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September 1, 1986 - August 31, 1988

INTERCONNECTION ALGORITHMS IN MULTI-HOP
PACKET RADIO TOPOLOGIES

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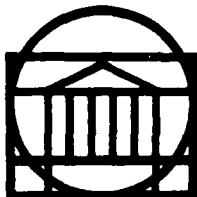800 N. Quincy Street
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SCHOOL OF ENGINEERING AND

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DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF VIRGINIA

CHARLOTTESVILLE, VIRGINIA 22901

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19. ABSTRACT We consider a two-cluster system in multi-hop packet radio topologies. Each cluster deploys a limited sensing random access algorithm, and contains local users who transmit their packets only via the algorithm in their own cluster. The system also contains marginal users, who may transmit their packets via either one of the algorithms in the two clusters.

For the above system, we adopt a limited sensing random access algorithm per cluster that has been previously studied. This algorithm utilizes binary, collision versus noncollision, feedback per slot, and in the presence of the limit Poisson user model and the absence of marginal users its throughput is 0.43. We consider a dynamic interconnection policy for the marginal users, and we then study the overall system performance in the presence of limit Poisson user populations. Specifically, we study the stability regions of the system and the per packet expected delays. Our interconnection policy accelerates the marginal users, presenting them with a significant delay advantage over the local users. This is desirable when the marginal users transmit high priority data, for example.

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I. INTRODUCTION

We consider multi-hop packet radio topologies, where environmental disturbances dictate the deployment of dynamic assignments for the transmission channels or frequencies. Then, dynamically reconfigurated cluster topologies result, and system connectivity is ensured via store-and-forward operations deployed by users located in cluster boundaries. At each point in time, the system induces a topology as that exhibited in Figure 1, where the same topology has been considered in [1]. In each cluster, a single common channel is used for transmissions by the ordinary nodes (or users) in it. The clusterhead in each cluster is one of the ordinary nodes (or users) and its function is to monitor the multiple access transmission algorithm in the cluster, and to provide the required feedbacks. The gateway nodes together with the clusterheads form the backbone network in the system, and ensure intracluster connectivity. The dynamics of the cluster formations and the structure of the backbone network are directly related to the operational and performance characteristics of the deployed multiple-access algorithms.

In the topologies considered here, the performance criteria include reliability, connectivity, economy, bandwidth efficiency, effective control of transmission delays, and robustness in the presence of channel errors. For reliability, economy, and bandwidth efficiency, the deployed multiple access algorithms should require very little knowledge about the system state by the users, since such knowledge can be obtained only at the expense of available bandwidth and its use requires complex and expensive hardware, and since the mobile users are generally unable to maintain overall system state information at all times. Thus, for the present topologies, we only consider Limited Sensing Random Access Algorithms (LSRAAs) for transmission, (as those in [3] and [4]). Those algorithms only require that each user know the algorithmic operations, can detect the transmission frequency of the cluster he is located in, and can sense the channel feedback from the time he generates a packet to the time that this packet is successfully transmitted; they do not require additional knowledge about the system state or topological information.

As the topology in Figure 1 evolves dynamically, and for better system connectivity, neighboring clusters may overlap (see [1], for example). The ordinary nodes located in the overlapping regions are then exposed to transmissions and feedbacks from more than one clusters; a generally time-varying phenomenon due to the mobility of the ordinary nodes, which can be exploited for the improvement in performance of the overall system, (as done in [2], for the mobile telephone system). Consider, for example, clusters 1 and 2 in Figure 1, and let us call the ordinary nodes in their overlapping region, marginal users; let us call the ordinary nodes in cluster i, i=1,2, which are not located in the overlapping region, local users for cluster i. The local users in cluster i communicate via the LSRAA deployed by the cluster, called LSRAAi, i=1,2. Due to the double exposure, the marginal users have a choice: For communication with each other and system connectivity, they can join either one of the LSRAA1 and LSRAA2 algorithmic systems. This choice can be implemented either dynamically, dictated by the two feedback sequences (from the two clusters) that the marginal users observe, or statically; the specifics of its implementation will be called, interconnection algorithm. A static interconnection algorithm is represented by a priori assigned probabilities. Then, upon generation of a new packet, each marginal user joins the LSRAAi with probability p_i, i=1,2, where $p_1 + p_2 = 1$, and remains there until his packet is successfully transmitted. The above static policy is simple, but requires that the marginal users know the a priori assigned probabilities at all times. In the dynamically changing topologies considered here, those probabilities should change dynamically and their values should remain known to all users. But this implies knowledge of the system dynamics at

all times, which is either very hard to obtain or requires a tremendous increase in feedback information, (and thus tremendous increase in the bandwidth of the feedback channels). Therefore, only dynamic interconnection algorithms are appropriate for the mobile multi-hop packet radio system. Such algorithms may in addition give an advantage to the marginal users, in terms of delays. This may be desirable in systems where the marginal users transmit either priority messages or control data, (including topological information for reconfiguration of the cluster structure, etc.).

In this paper, we consider two overlapping clusters, each deploying a LSRAA for its local users. Then, we propose a dynamic interconnection algorithm, via which marginal users join either one of the local per cluster LSRAA. For each of the two local LSRAAs, we adopt the limited sensing form of the algorithm in [4]. The latter algorithm has simple operational properties, in the presence of the limit Poisson user model it attains throughput 0.43, it has superior resistance to feedback errors, and operates with binary-collision versus noncollision-feedback.

The organization of the paper is as follows: In Section II, we present the system model. In Section III, we describe the local LSRAAs and the algorithm that the marginal users deploy. In Section IV, we present the algorithmic analysis of the two-cluster system. In Section V, we include comments and conclusions.

II. SYSTEM MODEL

We consider the two-cluster packet-radio system in Figure 1. We assume that in each of the two clusters, some synchronous LSRAA is deployed. In particular: (1) Time is divided in slots of length equal to the duration of a packet, and the starting instants of the slots are identical in both clusters. (2) In each cluster, the clusterhead broadcasts a feedback per slot, which corresponds to the outcomes induced by the local LSRAA. This feedback is either ternary, Collision (C) versus Success (S) versus Emptiness (E), or binary, Collision (C) versus Non-Collision (NC). (3) In each cluster, each local user is required to monitor the feedback from the local clusterhead continuously, from the time he generates a new packet to the time that his packet is successfully transmitted. We assume that no propagation delays and no forward or feedback channel errors exist in the system.

We assume that each marginal user receives the feedbacks from both the local clusterheads correctly and without propagation delays. At the time when a marginal user generates a new packet, he starts monitoring the feedbacks from both clusterheads continuously, until he decides to join the operations of one of the two LSRAAs, for the transmission of his packet. Upon this decision, he maintains the continuous monitoring of only those feedbacks that correspond to the LSRAA he chose, until his packet is successfully transmitted.

It is assumed that the mobility of the users in the system is low enough, so that each user remains within the same geographical region (local or marginal) from the time he generates a packet to the time that this packet is successfully transmitted. If the local LSRAAs in each cluster have good delay characteristics, then this time period may be relatively small, with high probability.

The local user populations in each cluster and the marginal user population are all modelled as limit Poisson. That is, it is assumed that the local traffic generated in cluster i, i=1, 2, is a Poisson process with intensity λ_i, i=1,2, and that the traffic generated by the marginal users is

another Poisson process with intensity λ_3. As found in [5], for a large class of LSRAAs, the limit Poisson user model provides a lower bound in performance, within the class of identical and independent users whose packet generating process is i.i.d.

III. THE ALGORITHMS

We assume that the two LSRAAs in the system are identical. Each LSRAA is the window algorithm in [4], which induces throughput 0.43 and operates with binary C versus NC feedback. This algorithm has simple properties and operations, while the algorithm in [3] is more complex. In addition, the algorithm in [4] has superior resistance to feedback errors.

Upon generation of a new packet, a marginal user imagines himself belonging to the systems of both the LSRAAs and follows their algorithmic steps, until the first time that he enters a collision resolution event in one of them. Then, he remains with the latter LSRAA system, until his packet is successfully transmitted.

In this paper, we consider the case where each local LSRAA is the limited sensing version of the two-cell algorithm in [4]. The reasons for this choice are several: This algorithm attains high enough throughput and induces low delays, while at the same time has simple operational and analytical properties, and is very resistant to feedback errors. For completeness, we describe the algorithm here.

Let time be measured in slot units, where slot t occupies the time interval [t, t+1). Let $x_t(j)$ denote the feedback that corresponds to slot t, for cluster j; j=1,2, where $x_t(j)$=C and $x_t(j)$=NC represent collision and noncollision slot t in cluster j, respectively. The local LSRAA in cluster j is implemented independently by each user in the system, and utilizes a window of length Δ. Let some local in cluster j user generate a new packet within the time interval $[t_1, t_1+1)$. Then, he immediately starts observing the feedback sequence $\{x_t(j)\}_{t\geq t_1}$, beginning with the feedback $x_{t_1}(j)$. Let us define the sequence $\{t_i(j)\}_{i\geq 2}$, as follows: $t_2(j)$ is the first time after t_1, such that $x_{t_2(j)}(j) = x_{t_2(j)-1} = NC$. Then, as will be explained below, $t_2(j)$ corresponds to the ending slot of a Collision Resolution Interval (CRI) in cluster j, and from $t_2(j)+1$ on, the user can identify the ending slots of CRIs induced by the algorithm in cluster j. Each $t_i(j)$ corresponds to the ending slot of some CRI in cluster j, and $t_{i+1}(j)$ is the first after $t_i(j)$ such slot. At $t_i(j)$, the user updates his arrival instant, as follows: $t^{(i)} = t_1 + (i-2)\Delta$; we call the sequence $\{t^{(i)}\}_{i\geq 2}$, updates. Let $t_k(j)$ be such that: $t_k(j)\epsilon\{t_i(j)\}_{i\geq 2}$, $t^{(i)}<t_i(j)-1-\Delta$; $\forall i\leq k-1$, and $t^{(k)}>t_k(j)-1-\Delta$. Then, in slot $t_k(j)+1$, the user enters a CRI within the LSRAA of cluster j, and transmits his packet successfully during its process. He stops observing the feedback sequence $\{x_t(j)\}$ at the point when his packet is sucessfully transmitted. If the user is instead marginal, then he observes both feedback sequences $\{x_t(1)\}_{t\geq t_1}$ and $\{x_t(2)\}_{t\geq t_1}$, and follows the evolution of both the time sequences $\{t(1)\}_{i\geq 2}$ and $\{t_i(2)\}_{i\geq 2}$. If $t_k(1)<t_k(2)$, then in slot $t_k(1)+1$ he enters a CRI within the LSRAA of cluster 1, and transmits his packet successfully during its process. If $t_k(1)>t_k(2)$, instead, then he joins a CRI in cluster 2, in slot $t_k(2)+1$. If $t_k(1)=t_k(2)$, then he selects one of the local LSRAAs with probability 0.5. The above, describe the first entry rules, for the local and the marginal users; that is, how and when each newly generated packet first starts participating in some CRI, for its successful transmission. From the first entry rule that the marginal users utilize, it is clear that they have an advantage over the local for clusters 1 and 2 users. In particular, their waiting time until they first enter some CRI is generally smaller than that of the local users; thus, their

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overall delays are generally smaller than those of the local users as well.

Consider the algorithm in cluster j, and let it start operating at time zero. Then, slot 1 is empty. In slot 2, the arrivals in [0,1) are transmitted, and a CRI begins. If the number of arrivals in [0,1) is less than two, then $x_2(j)=NC$, the CRI lasts one slot, and a new CRI begins with slot 3. If the number of arrivals in [0,1) is at least two, then $x_2(j)=C$, instead, and the CRI lasts as long as it takes to resolve the collision in slot 2; its end is identifiable by all the users in the system, (as will be seen below). In general, let T be a slot that corresponds to the end of some CRI. Then, in slot T+1, all the users with current updates in $(T-\Delta-1, T-1]$ transmit. If $x_{T+1}(j)=NC$, then the CRI which started with slot T+1 lasts one slot, and a new CRI starts with slot T+2. If $x_{T+1}(j)=C$, instead, then a collision occurs, whose resolution starts with slot T+2. No arrivals that did not participate in the collision at T+1 are transmitted, until the latter is resolved. During the collision resolution, each involved user acts independently, via the utilization of a counter whose value at time t is denoted r_t. The counter values can be either 1 or 2, and they are updated and utilized according to the rules below.

1. The user tranmits in slot t, if and only if $r_t=1$. A packet is successfully transmitted in t, if and only if $r_t=1$ and $x_t=NC$.

2. The counter values transition in time as follows:

(a) If $x_{t-1}=NC$ and $r_{t-1}=2$, then $r_t=1$

(b) If $x_{t-1}=C$ and $r_{t-1}=2$, then $r_t=2$

(c) If $x_{t-1}=C$ and $r_{t-1}=1$, then

$$r_t = \begin{cases} 1 \text{ , with probability 0.5} \\ 2 \text{ , with probability 0.5} \end{cases}$$

A CRI which starts with a collision, ends when it becomes known to all users that the initially collided packets have been successfully transmitted. From the operations exhibited above, it is not hard to see that such a CRI ends the first time (after its beginning) that two consecutive NC slots occur.

IV. ALGORITHMIC ANALYSIS

For convenience in notation, we will refer to the local users in cluster 1, the local users in cluster 2, and the marginal users, as subsystem 1, subsystem 2, and subsystem 3, respectively. In the algorithmic analysis, we will adopt the limit Poisson user model (infinitely many independent Bernoulli users) for each of the three subsystems. In particular, we will assume that the three subsystem traffics are mutually independent, and that the user traffic in subsystem j, j=1,2,3, is limit Poisson with intensity λ_j.

Consider either one of the local LSRAAs, whose operations are described in section III. Consider some CRI within the system of the LSRAA, which starts with transmissions from packet arrivals in an arrival interval of cumulative length, u. We will call u, the "length of the examined interval." Let us then define:

$E\{l | u\}$: Given length of the examined interval equal to u, the expected number of slots needed for its resolution; that is, for the successful transmission of all the arrivals in the examined interval.

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$L_{n,k-n}$:   Given that n packets have counter values equal to 1, and k-n packets have counter values equal to 2, the expected number of slots needed by the local LSRAA, for the successful transmission of all the k packets.

Then, directly from the results in [4], we have:

$$L_{0,0} = L_{1,0} = 1 \ , \ L_{0,i} = 1 + L_{i,0} \ ; \ i \geq 1$$

(1)

$$0 < L_{k,0} \leq \frac{3}{4}k^2 + \frac{9}{4}k - 2 \ ; \ k \geq 1$$

If the arrivals in the examined interval are controlled by a limit Poisson process whose intensity is λ, then:

$$E\{l \mid u\} \overset{\Delta}{=} E_\lambda\{l \mid u\} = \sum_{k=0}^{\infty} L_{k,0} \ e^{-\lambda u} \frac{(\lambda u)^k}{k!}$$

(2)

As found in [4], in the presence of the limit Poisson user model, each local LSRAA has throughput 0.43, attained for window size $\Delta^* = 2.33$. Thus, in view of the system considered in this paper, the Poisson intensities λ_j, j=1,2,3, must satisfy the following necessary conditions, for overall system stability:

$$\lambda_1 < 0.43, \text{ and } \lambda_2 < 0.43, \text{ and } \lambda_1 + \lambda_2 + \lambda_3 < 2(0.43) = 0.86$$

(3)

The necessary conditions in (3) determine a $\{\lambda_j, j=1,2,3\}$ hyperplane, which contains the $\{\lambda_j, j=1,2,3\}$ region that determines the system throughput. Tight bounds on the $\{\lambda_j, j=1,2,3\}$ space which provides the system throughput, will be attained via the system stability analysis in Section IV.1 below.

IV.1. System Stability

We consider the evolution of the algorithms in the two-cluster system, and we assume that the system starts operating at time zero. Let us consider the sequence in time of the CRIs induced by the two LSRAAs in the system. Let the sequence $\{T_n\}_{n \geq 0}$ be such that: (1) For each n, T_n corresponds to the starting point of a slot which is the beginning of some CRI. We note that at T_n, two CRIs may simultaneously begin; one for each of the two LSRAAs in the system. (2) T_n is the first after T_{n-1} time instant which corresponds to the beginning of some CRI. (3) $T_0=2$, and at T_0 two CRIs begin; one for each of the two LSRAAs in the system.

Let $\{T_n^{(s)}\}_{n \geq 0}$ be the subsequence of sequence $\{T_n\}_{n \geq 0}$, which consists of those time instants when two CRIs begin simultaneously; one for each of the two LSRAAs in the system. Clearly, $T_0^{(s)} = T_0 = 2$. Let $D_{n,s}^{(j)} + 1$, j=1,2,3, denote the total length of the unresolved arrival intervals in subsystem j, at the time instant $T_n^{(s)}$. $D_{n,s}^{(j)}$ is then called "the lag of subsystem j at time $T_n^{(s)}$." From the algorithmic operations in the system, we conclude: (1) $D_{n,s}^{(j)} \geq 1$ and the values of $D_{n,s}^{(j)}$ are denumerable for all n and j. (2) $D_{0,s}^{(j)} = 1$, j=1,2,3. (3) At time $T_n^{(s)}$, the LSRAA in cluster k, k=1,2, examines two arrival intervals: one from subsystem k which has length $\min(D_{n,s}^{(k)}, \Delta)$ and contains arrivals generated by a Poisson process with intensity λ_k, and one from subsystem 3 which has length $\min(D_{n,s}^{(3)}, \Delta)$ and contains arrivals generated by a Poisson process with intensity $0.5 \ \lambda_3$. (4) The triple $(D_{n,s}^{(j)}, j=1,2,3)$ describes the state of the system at time $T_n^{(s)}$, and the sequence $\{S_n\}_{n \geq 0} \overset{\Delta}{=} \{D_{n,s}^{(j)}, j=1,2,3\}_{n \geq 0}$ is a three-dimensional irreducible and

aperiodic Markov Chain.

The stability of the system is represented by the ergodicity of the three-dimensional Markov Chain $\{S_n\}_{n\geq 0}$. In addition, at time $T_n^{(s)}$, the backlogs of each of the two LSRAAS and of the overall system are all respresented by the three lags, $D_{n,s}^{(j)}$, $j=1,2,3$. Given $D_{n,s}^{(j)} = d_n^{(j)}$, $j=1,2,3$, the expected system backlog at time $T_n^{(s)}$ is $\sum_{j=1}^{3} \lambda_j d_n^{(j)}$, where $V(\overline{d}_n) = V(\{d_n^{(j)}\}_{1\leq j\leq 3}) \overset{\Delta}{=} \sum_{j=1}^{3} \lambda_j\, d_n^{(j)}$ is a Lyapunov function of the three system lags $\{d_n^{(j)}\}_{1\leq j\leq 3}$. Let C denote the state space of the Markov Chain $\{S_n\}_{n\geq 0}$, and let us then define the operator $AV(\overline{d}) = AV(\{d^{(j)}\}_{1\leq j\leq 3})$, called a generalized drift, as follows:

$$AV(\overline{d}) = AV(\{d^{(j)}\}_{1\leq j\leq 3}) \overset{\Delta}{=} E\{V(\{D_{n+1,s}^{(j)}\}_{1\leq j\leq 3}) - V(\{D_{n,s}^{(j)}\}_{1\leq j\leq 3}) \mid D_{n,s}^{(j)} = d^{(j)}, j=1,2,3\}$$

$$= E\{\sum_{j=1}^{3} \lambda_j D_{n+1,s}^{(j)} \mid D_{n,s}^{(j)} = d^{(j)}, j=1,2,3\} - \sum_{j=1}^{3} \lambda_j d^{(j)}; \; \overline{d} = \{d^{(j)}\}_{1\leq j\leq 3} \in C \tag{4}$$

The generalized drift in (4) can be used to establish necessary and sufficient conditions for the ergodicity of the Markov Chain $\{S_n\}_{n\geq 0}$. To see that, let us define:

$$p_{\overline{d},\overline{e}} \overset{\Delta}{=} P(\overline{d}_{n+1} = \overline{e} \mid \overline{d}_n = \overline{d}) \tag{5}$$

$$F_{\overline{d},V}(z) \overset{\Delta}{=} (1-z)^{-1}\left[z^{V(\overline{d})} - \sum_{\overline{e}\in C} p_{\overline{d},\overline{e}}\, z^{V(\overline{e})} \right];$$

$$; z \in [0,1), \; \overline{d} \in C \tag{6}$$

; where $F_{\overline{d},V}(z)$ is the generalized Kaplan function on the Lyapunov function V, [11]. From the results in [7], [8], [9], and [10], we can then express the following Proposition.

<u>Proposition</u>

If it can be established that:

(i) $|AV(\overline{d})| < \infty$, for all $\overline{d}$ in C. \hfill (7)

(ii)   There exist finite subset H_2 of C and some nonnegative finite constant B, such that,

$$F_{\overline{d},V}(z) \geq -B, \text{ for all } z \text{ in}[0,1), \text{ and all } \overline{d} \text{ in } C-H_2 \tag{8}$$

Then,

(A)   If there exists finite subset H_1 of C, and some $\varepsilon' > 0$, such that,

$$AV(\overline{d}) < -\varepsilon', \text{ for all } \overline{d} \text{ in } C-H_1 \tag{9}$$

then, the Markov Chain $\{S_n\}_{n\geq 0}$ is ergodic.

(B)   If H is a proper subset of C, such that,

$$\inf_{\overline{d}\in[C-H]} V(\overline{d}) > \sup_{\overline{d}\in H} V(\overline{d}) \tag{10}$$

$$AV(\bar{d}) \geq 0, \text{ for all } \bar{d} \text{ in } C-H \qquad (11)$$

then, the Markov Chain $\{S_n\}_{n \geq 0}$ is nonergodic. $\square$

Remark. Condition (8) is trivially satisfied for Markov Chains which are downward uniformly bounded with respect to the Lyapunov function V; that is, for Chains such that, $P(V(\bar{D}_{n+1}) < V(\bar{d}_n) - x) = 0$; for all $x > \alpha$, for some positive and finite constant α, and for all $\bar{d}_n$ in C. In the problem studied in this paper, when $\lambda_3 = 0$, two decoupled systems arise. Each of those systems corresponds then to a local LSRAA, whose lags at collision resolution points form a downward uniformly bounded Markov Chain; the Lyapunov function $V(\bar{d})$ reduces then to $d^{(j)}$, for the system in cluster j, j=1,2, and the constant α equals then $\lfloor \Delta \rfloor = 2$, (see [4]).

We now express a lemma, whose proof is in the Appendix.

Lemma 1

Consider the Markov Chain $\{S_n\}_{n \geq 0} = \{D_{n,s}^{(j)}, j=1,2,3\}_{n \geq 0}$ and its Lyapunov function $V(\bar{d}_n) = \sum_{1 \leq j \leq 3} \lambda_j d_n^{(j)}$, and let C be the state space of $\{S_n\}_{n \geq 0}$. Let $AV(\bar{d})$ and $F_{\bar{d},V}(z)$ be as in (4) and (6), respectively. Then,

(i) $|AV(\bar{d})| < \infty$, for all $\bar{d}$ in C and all $\{\lambda_j\}: \sum_{j=1}^{3} \lambda_j \leq 1$ $\qquad (12)$

(ii)   There exist positive finite constant B and some finite subset H_2 of C, such that:

$$F_{\bar{d},V}(z) \geq -B, \text{ for all } z \text{ in}[0,1), \text{ all } \bar{d} \text{ in } C-H_2, \text{ and all } \{\lambda_j\}: \sum_{j=1}^{3} \lambda_j \leq 1 \quad \square \qquad (13)$$

Due to Lemma 1, and in view of the Proposition, the search for the ergodicity versus the nonergodicity of the Markov Chain $\{S_n\}_{n \geq 0}$ reduces to the identification of appropriate properties, for the satisfaction of conditions (A) and (B) in the Proposition, respectively.

Consider $T_n^{(s)}$, and let the lags $\{D_{n,s}^{(j)}\}_{1 \leq j \leq 3}$ be sufficiently long, so that in the time interval $[T_n^{(s)}, T_{n+1}^{(s)}]$, each CRI from each of the two LSRAAs in the system, resolves an arrival interval of length Δ, (as proven in the Appendix, the lengths of the lags $\{D_{n,s}^{(j)}\}_{1 \leq j \leq 3}$ that satisfy this con-_ution are finite with probability close to one). Given system Poisson rates $\{\lambda_j\}_{1 \leq j \leq 3}$, and for $E_\lambda\{l|u\}$ as in (2), let us then define:

i=1,2 ; $N_i(\{\lambda_j\},\Delta)$: Given $\{\lambda_j\}$, the number of CRIs generated by the algorithm in cluster i, in $[T_n^{(s)}, T_{n+1}^{(s)}]$, when each CRI resolves arrival intervals of length Δ, from both its local and the marginal users. The first such CRI starts at $T_n^{(s)}$, and the last such CRI ends at $T_{n+1}^{(s)}$.

$AV(\bar{d}_\Delta, \{\lambda_j\})$: The generalized drift in (4), for given Poisson rates $\{\lambda_j\}$, when, in $[T_n^{(s)}, T_{n+1}^{(s)}]$, each CRI generated by either algorithm in the system, resolves arrival intervals of length Δ.

$$i=1,2; \; A^{(i)}V(\bar{d}_\Delta, \{\lambda_j\}) \overset{\Delta}{=} E_{\lambda_i + \lambda_3 | 2}\{l|\Delta\} - \Delta +$$

$$+ \left[E\{N_i(\{\lambda_j\},\Delta)\} - 1 \right] \left[E_{\lambda_i + \lambda_3}\{l|\Delta\} - \Delta \right] \qquad (14)$$

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Then, via (4), we easily derive the following expression:

$$AV(\overline{d}_\Delta, \{\lambda_j\}) = (\lambda_1 + \frac{\lambda_3}{2})A^{(1)}V(\overline{d}_\Delta, \{\lambda_j\}) + (\lambda_2 + \frac{\lambda_3}{2})A^{(2)}V(\overline{d}_\Delta, \{\lambda_j\}) +$$

$$- \frac{\lambda_3 \Delta}{2}\Big[E\{N_1(\{\lambda_j\}, \Delta)\} + E\{N_2(\{\lambda_j\}, \Delta)\} - 2\Big] \tag{15}$$

; where,

$$E_{\lambda_1 + \lambda_3 | 2}\{l | \Delta\} + [E\{N_1(\{\lambda_j\}, \Delta)\} - 1]E_{\lambda_1 + \lambda_3}\{l | \Delta\} =$$

$$= E_{\lambda_2 + \lambda_3 | 2}\{l | \Delta\} + [E\{N_2(\{\lambda_j\}, \Delta)\} - 1]E_{\lambda_2 + \lambda_3}\{l | \Delta\} \tag{16}$$

We note that $A^{(i)}V(\overline{d}_\Delta, \{\lambda_j\})$ in (14) is a generalized drift for the algorithm in cluster i. We now state a theorem whose proof is in the Appendix.

Theorem 1

(i)   Let there exist some $\varepsilon > 0$, such that the two conditions below are satisfied:

$$A^{(1)}V(\overline{d}_\Delta, \{\lambda_j\}) < -\varepsilon$$

$$A^{(2)}V(\overline{d}_\Delta, \{\lambda_j\}) < -\varepsilon \tag{17}$$

Then, the Markov Chain $\{S_n\}_{n \geq 0}$ is ergodic at the Poisson rates $\{\lambda_j\}_{1 \leq j \leq 3}$.

(ii) Let at least one of the two generalized drifts, $A^{(i)}V(\overline{d}_\Delta, \{\lambda_j\})$, i=1,2, be nonnegative. Then, the Markov Chain $\{S_n\}_{n \geq 0}$ is nonergodic at the Poisson rates $\{\lambda_j\}_{1 \leq j \leq 3}$. □

Remark: The stability analysis remains unchanged, when the two random access algorithms, in clusters 1 and 2, are full feedback sensing, instead. The full versus limited feedback sensing choice affects only the transmission delays.

The conditions (17) in the Theorem define lower bounds on the $\{\lambda_j\}$ regions for which the Markov Chain $\{S_n\}_{n \geq 0}$ is ergodic. Since the constant ε in (17) can be arbitrarily small, and in combination with the statement in part (ii) of the Theorem, we conclude that the ergodicity bounds determined by (17) are tight.

In the Appendix, and in Section VI.2 in particular, we describe the methodology regarding the computation of the $\{\lambda_j\}$ values for which the Chain $\{S_n\}_{n \geq 0}$ is ergodic, as well as the pertinent quantities and their recursions. In Table 1, we include the maximum rate λ_2^* accepted by the system, for varying λ_1 and λ_3 values, as well as the maximum accepted symmetric rate $\lambda^*(\lambda_3) = \lambda^* = \lambda_1^* = \lambda_2^*$, for varying λ_3 values. We also include the maximum rate λ_3^* accepted by the system, when $\lambda_1 = \lambda_2 = 0$. We note that due to symmetries appearing in the system when λ_1 and λ_2 are interchanged, for each fixed rate λ_3, we only need to compute λ_2^* for λ_1 values in the interval $[0, \lambda^*(\lambda_3))$. In Figure 2, we plot the boundaries of the (λ_1, λ_2) acceptable regions, parametrized by various λ_3 values. Those boundaries are clearly symmetric around the 45° straight line. In Figure 3, we plot λ_3^* against $\lambda = \lambda_1 = \lambda_2$, for symmetric two-cluster systems.

From Table 1 and Figures 2 and 3, we observe that whenever λ_3 is strictly positive, then the sum $\lambda_1^* + \lambda_2^* + \lambda_3$ is strictly less than 0.86; that is, strictly less than twice the throughput of each local LSRAA. This is so, because when $\lambda_3 > 0$, some percentage of the traffic generated by the

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marginal users is always assigned for transmission to an even overloaded by local traffic LSRAA. Thus, if the rate of the local traffic is close to the algorithmic throughput, then the presence of marginal users results in instability of the LSRAA.

IV.2. System Delays

The two-cluster algorithmic system induces regenerative points within its stability region. Thus, in essence, the methodology in [6] applies for the computation of the expected delays, in each of the three subsystems. To see that, consider the sequence $\{S_n\}_{n \geq 0} = \{D_{n,s}^{(j)}, j=1,2,3\}_{n \geq 0}$ of lag vectors induced by the algorithmic system, at the time instants $\{T_n^{(s)}\}_{n \geq 0}$. As established before, the lag vector S_0 is the unity vector. Let us define the sequence $\{R_i\}_{i \geq 1}$ as follows: $R_1 = T_0^{(s)} = 2$, and each R_i corresponds to some $T_n^{(s)}$ instant such that the lag vector S_n is the unity vector; R_{i+1} is the first after R_i such instant. Let $Q_i^{(j)}$, $i \geq 1$, $j=1,2,3$, denote the number of transmitted packets from subsystem j, in the time interval $(0, R_i]$. Then, $G_i^{(j)} \overset{\Delta}{=} Q_{i+1}^{(j)} - Q_i^{(j)}$ denotes the number of transmitted packets from subsystem j, in the time interval $(R_i, R_{i+1}]$, and for memoryless traffics, such as the Poisson, the sequences $\{G_i^{(j)}\}_{i \geq 1}$, $j=1,2,3$, are sequences of i.i.d. random variables; thus, the sequences $\{Q_i^{(j)}\}_{i \geq 1}$, $j=1,2,3$, are then renewal processes. In addition, if $D_n^{(j)}$ denotes the delay experienced by the n-th transmitted packet arrival from subsystem j, then, the delay process $\{D_n^{(j)}\}_{n \geq 1}$, $j=1,2,3$, induced by the algorithmic system is regenerative with respect to the process $\{Q_i^{(j)}\}_{i \geq 1}$, and the process $\{G_i^{(j)}\}_{i \geq 1}$ is nonperiodic for every j, since $P(G_i^{(j)}=1)>0$, $j=1,2,3$. Let $D^{(j)}$ denote the expected steady-state delay experienced by a packet in subsystem j, $j=1,2,3$, and let us define:

$$Z^{(j)} \overset{\Delta}{=} E\{G_1^{(j)}\} \; , \; j=1,2,3$$

$$W^{(j)} \overset{\Delta}{=} E\{\sum_{i=1}^{G_1^{(j)}} D_i^{(j)}\} \; , \; j=1,2,3 \tag{18}$$

$$H \overset{\Delta}{=} E\{R_2 - R_1\}$$

Then, from the regenerative arguments in [6], we conclude:

$$D^{(j)} = W^{(j)} \, [Z^{(j)}]^{-1} \, , \, j=1,2,3 \tag{19}$$

$$\text{where,} \; Z^{(j)} = \lambda_j \, H \, , \, j=1,2,3 \tag{20}$$

The computation of bounds on the expected delays $D^{(j)}$, depend on the computation of tight upper and lower bounds on the quantities in (18). The pertinent quantities towards that direction and their recursions are included in the Appendix. We used them together with the methodology in [6] to compute expected per packet delays. We present some of our results in Figures 4, 5, 6, and 7. In Figures 4 and 5, we plot expected per packet delays, for two symmetric systems ($\lambda_1 = \lambda_2$), as functions of the Poisson intensity λ_3. In Figures 6 and 7, we plot expected delays against λ_3, for two asymmetric cases. From the four figures, we observe the advantage of the marginal users, in terms of expected delays. Even when the expected delays of the local users approach those that correspond to the throughputs of the LSRAAs, the expected delays of the marginal users remain low, never exceeding ten slots, for all the examined cases. The delay

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advantage of the marginal users, as compared to the local users, increases monotonically, as the rate of their traffic increases.

V. COMMENTS AND CONCLUSIONS

In this paper, we studied a two-cluster interconnected system. Each cluster deploys the limited sensing random access algorithm in [4], and the interconnection is due to marginal users, who dynamically select one of the two algorithms for their transmissions. We performed rigorous analysis of the overall system stability, and the expected per packet delays. Our stability analysis generalizes easily to multi-cluster systems, with marginal users who may select either one of the local algorithms for their packet transmissions. For M clusters, the ergodicity conditions are as those in Theorem 1, only that then there are M inequalities in (17) with indices varying from 1 to M. Then, when a marginal user with a packet to transmit observes simultaneous beginnings of K CRIs, (from the algorithms in K clusters, where $2 \leq K \leq M$), and is within the examined intervals of all of them, he selects each one of those CRIs with probability 1/K.

The interconnection policy adopted is dynamic, and requires no a priori knowledge of the traffic populations and characteristics, and of the states of the involved subsystems. It only requires knowledge of the algorithmic rules, and monitoring of feedbacks from the time a packet is generated to the time that it is successfully transmitted. In addition, the adopted interconnection policy presents a significant delay advantage to the marginal users. In all cases, it maintains the value of the expected per marginal packet delay below ten slots, even when the expected per local packet delays approach their limit values within the stability region of the system. This delay advantage to the marginal users may be of high importance, when they transmit high priority data, and when dynamic cluster reconfigurations may result in temporary isolation of the marginal users if the transmission of their data is delayed.
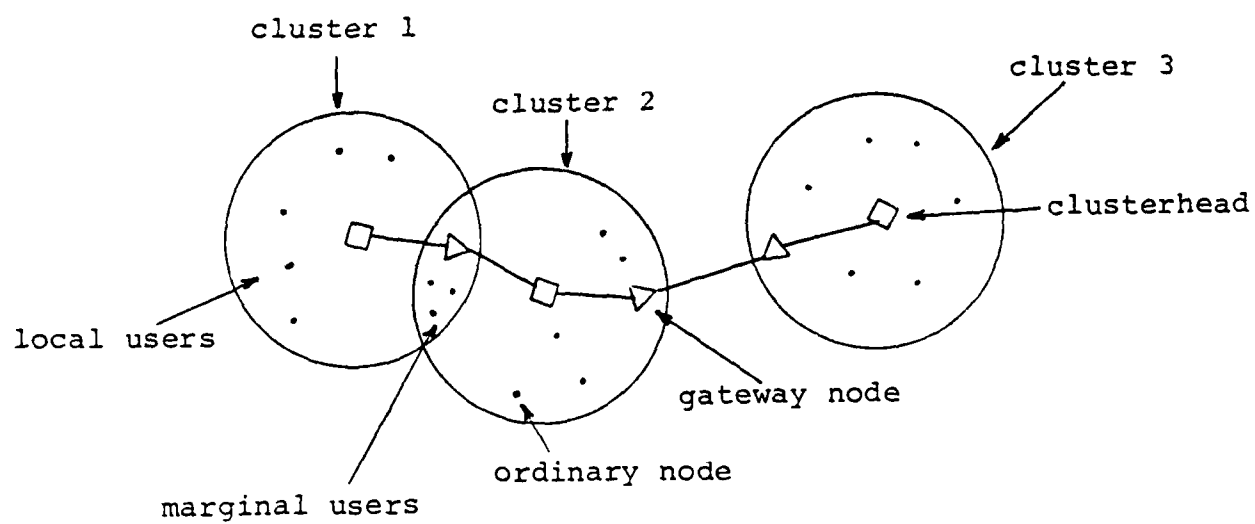
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Figure 1

Cluster Topology

λ_3	λ_1	λ_2^*	$\lambda^* = \lambda_1^* = \lambda_2^*$
0	< 0.43	0.43	0.43
0.001	0.01 0.10 0.20 0.30 0.40	0.429 0.4288 0.4285 0.4282 0.4280	0.4278
0.01	0.00 0.20 0.40	0.424 0.423 0.422	0.42
0.10	0.00 0.10 0.20 0.30	0.372 0.370 0.368 0.364	0.36
0.20	0.00 0.10 0.20 0.30	0.32 0.31 0.30 0.28	0.29
0.30	0.00 0.10 0.15 0.20	0.260 0.245 0.240 0.232	0.23
0.40	0.00 0.05 0.10 0.15	0.198 0.190 0.180 0.170	0.17
0.50	0.00 0.05 0.10	0.132 0.12 0.11	0.11
0.60	0.00 0.01 0.02	0.062 0.06 0.05	0.04

For $\lambda_1 = \lambda_2 = 0$: $\lambda_3^* = 0.68$

Table 1

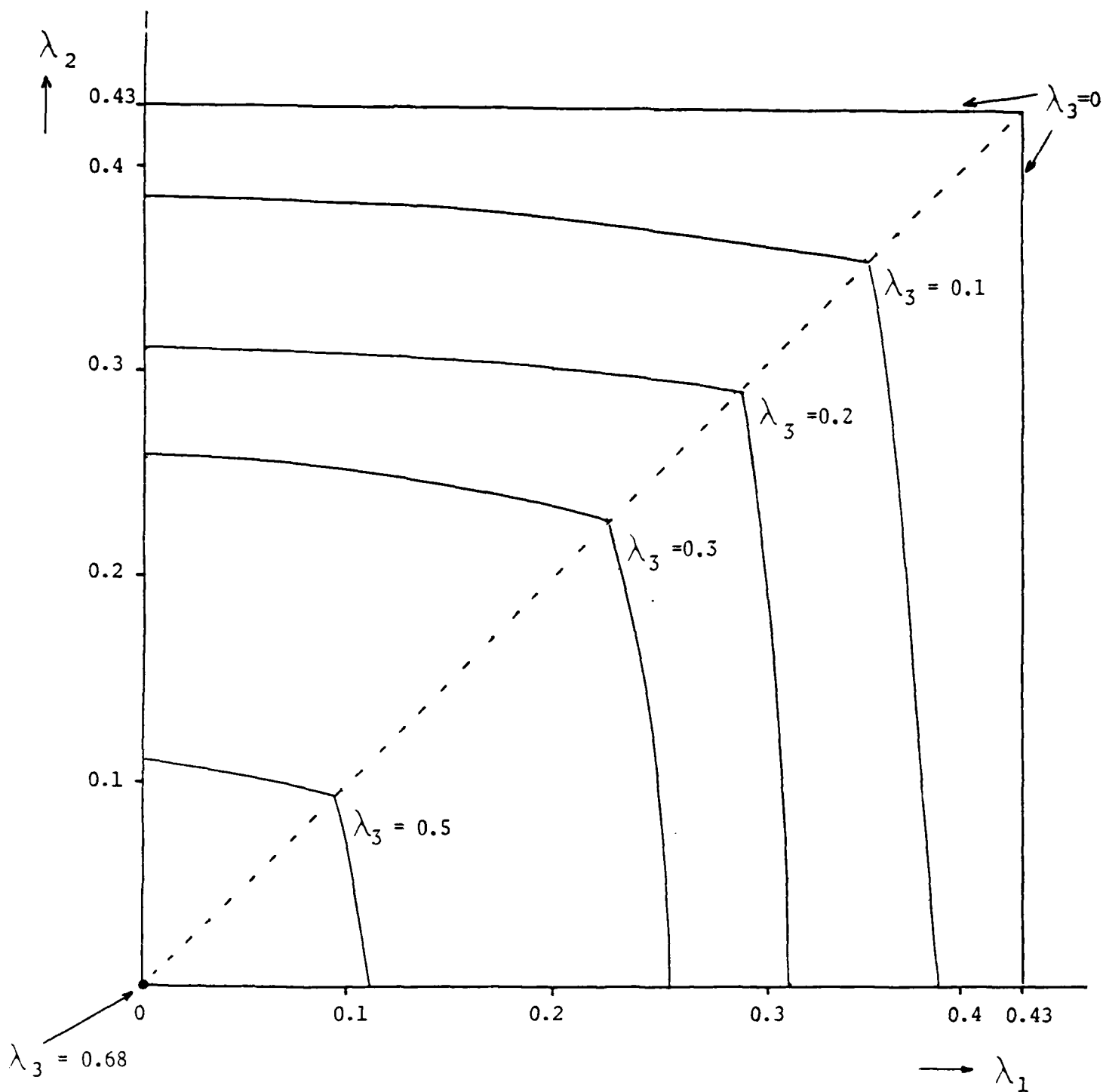Maximum Accepted Poisson Rates

Window Size: $\Delta = 2.33$

Figure 2

Boundaries of the (λ_1, λ_2) acceptable regions
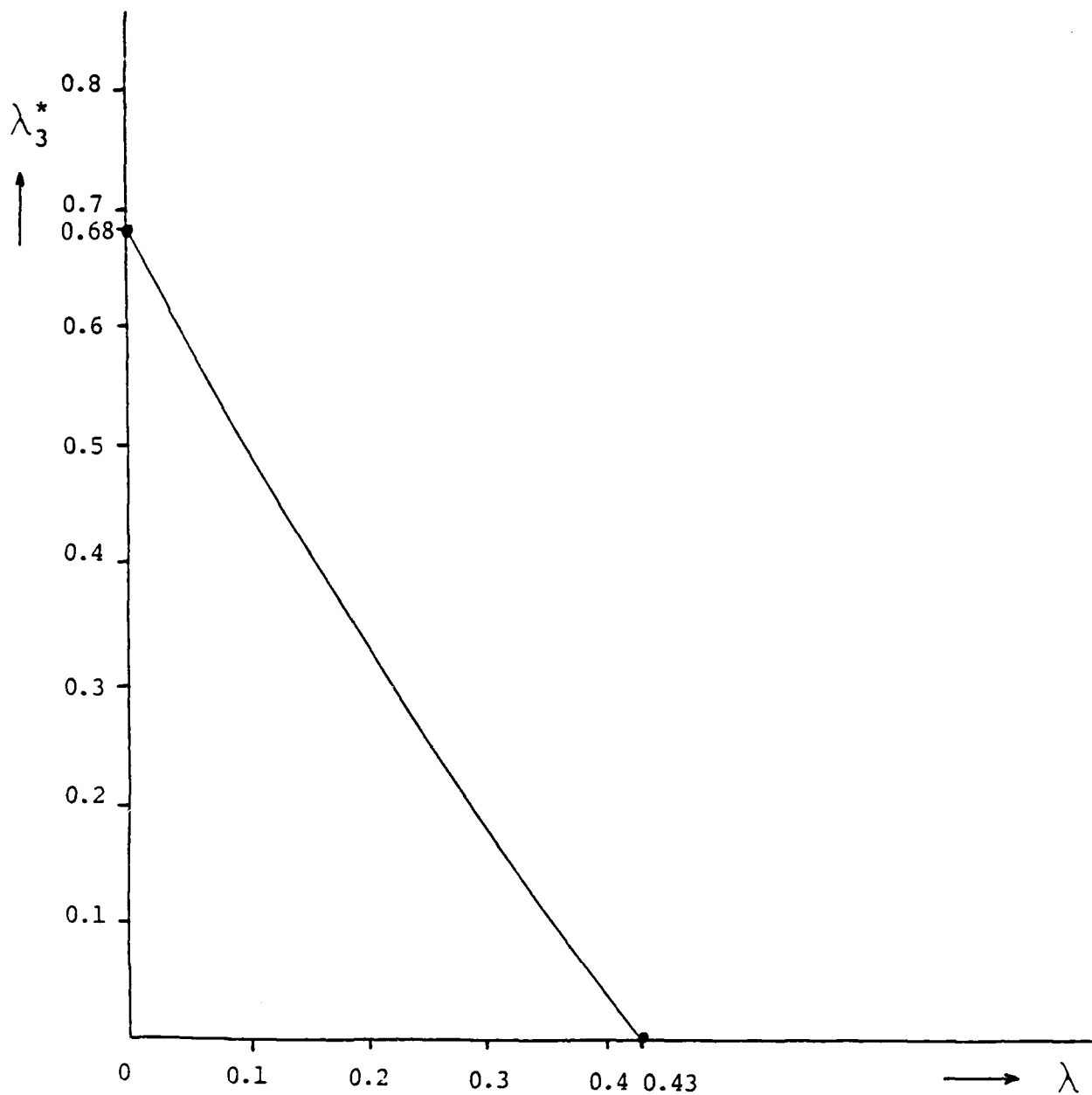parametrized by λ_3

Figure 3

Maximum acceptable rate λ_3^*, against $\lambda = \lambda_1 = \lambda_2$

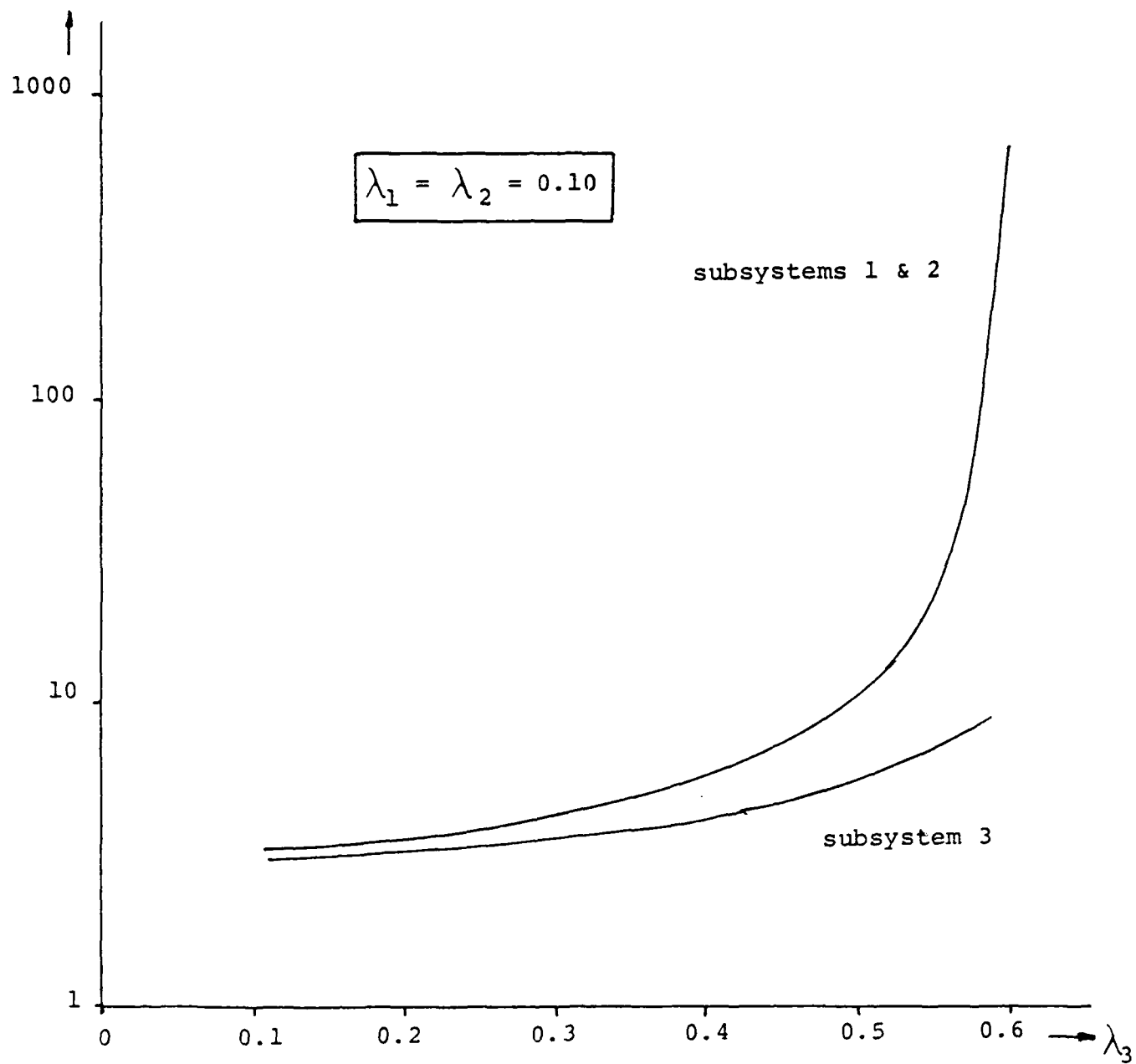Figure 4

Expected Per Packet Delays

Figure 5

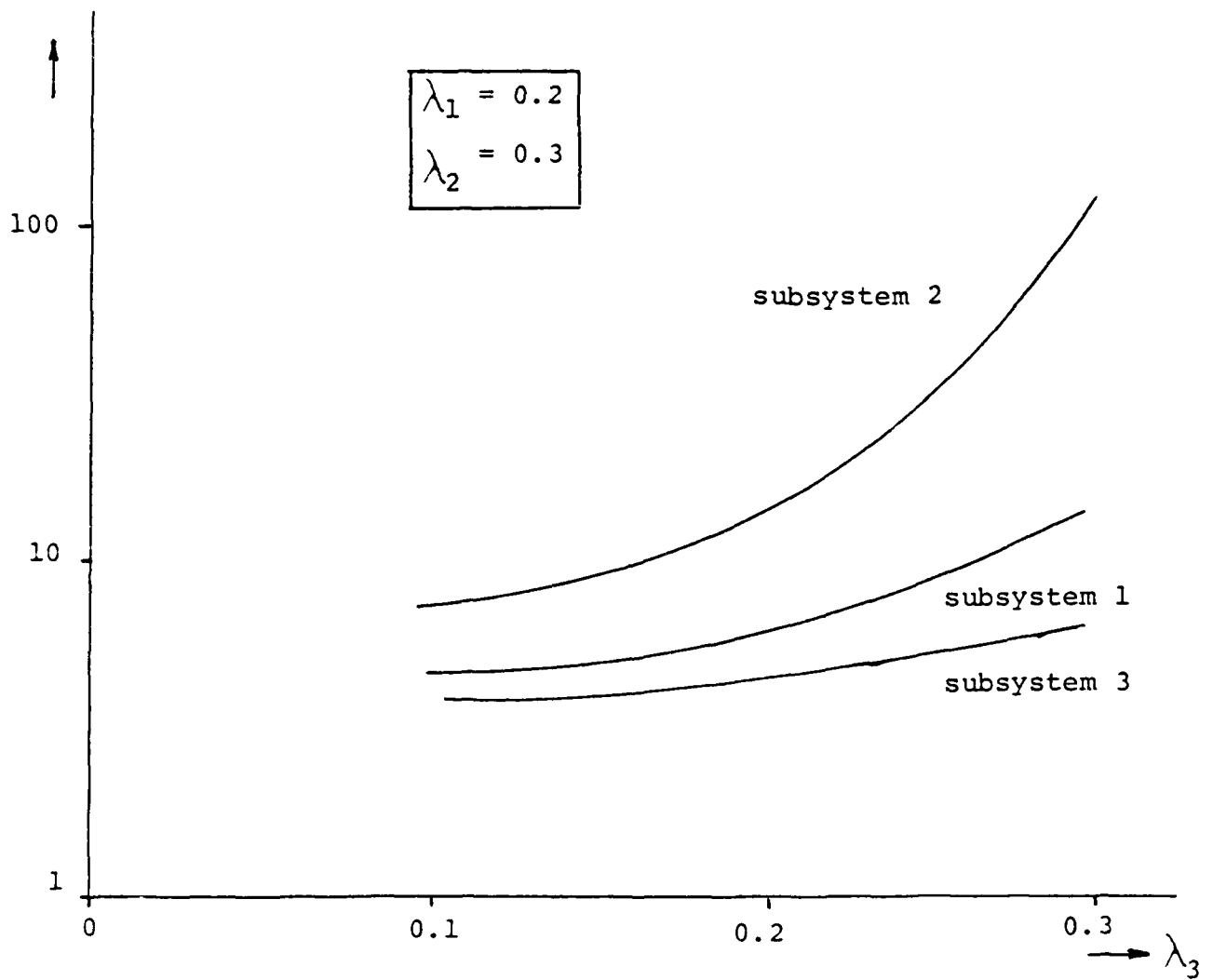Expected Per Packet Delays

$\lambda_1 = 0.2$
$\lambda_2 = 0.3$

subsystem 2

subsystem 1

subsystem 3

Figure 6

Expected Per Packet Delays

$\lambda_1 = 0.3$
$\lambda_2 = 0.1$

subsystem 1
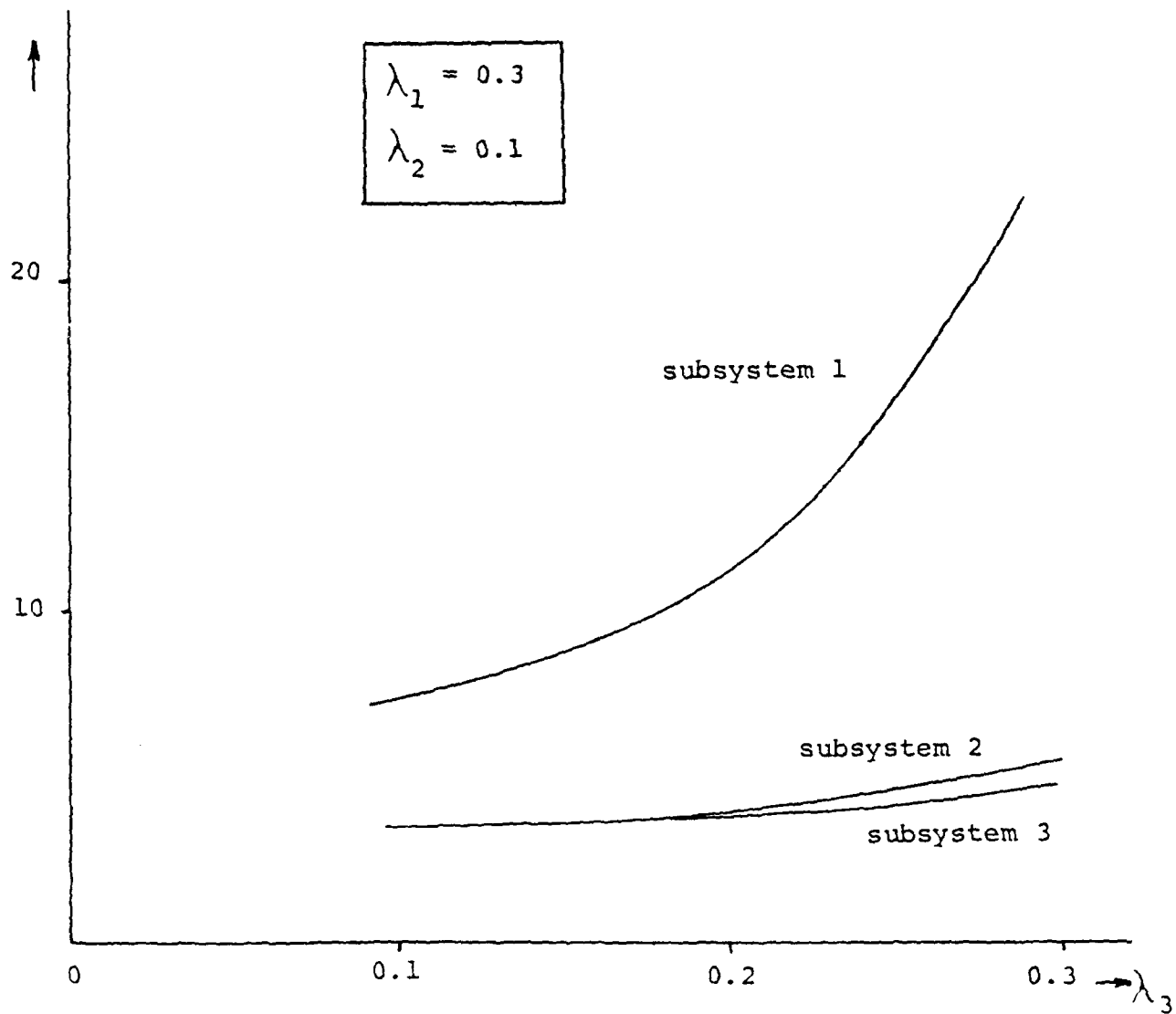
subsystem 2

subsystem 3

Figure 7

Expected Per Packet Delays

VI. APPENDIX

VI.1. Fundamental Results

We start by proving some fundamental algorithmic properties of the system, which will be used for the proof of the theorem and the lemma in the main part of the paper.

Let us define the following quantities:

N_i: The number of CRIs generated by the algorithm in cluster i, between the time instants $T_n^{(s)}$ and $T_{n+1}^{(s)}$, where i=1,2. The first such CRI starts at $T_n^{(s)}$, and the last such CRI ends at $T_{n+1}^{(s)}$.

$\delta_j^{(i)}(k)$: The length of the arrival interval from subsystem k, k=1,2,3, resolved by the j-th CRI generated by the algorithm in cluster i, i=1,2, between the time points $T_n^{(s)}$ and $T_{n+1}^{(s)}$. We note that $1 \leq \delta_j^{(i)}(k) \leq \Delta$; $\forall$ i, j, k.

$l(\lambda\delta_1 + \mu\delta_2)$: The length of a CRI, generated by either one of the two local LSRAAs, which resolves a length δ_1 interval containing arrivals from an intensity λ Poisson process, and a length δ_2 interval containing arrivals from an intensity μ Poisson process.

$$L_{1,\bar{d}}^{(i)} \overset{\Delta}{=} E\{l(\lambda_i\delta_1^{(i)}(i) + \frac{\lambda_3}{2}\delta_1^{(i)}(3)) \,|\, \bar{d}\} \; ; \; i=1,2$$

$$L_{j,\bar{d}}^{(i)} \overset{\Delta}{=} E\{l(\lambda_i\delta_j^{(i)}(i) + \lambda_3\,\delta_j^{(i)}(3)) \,|\, \bar{d}\} \; ; \; i=1,2 \,, \; j \geq 2 \tag{A.1}$$

$$E_{j,\bar{d}}^{(i)}(k) \overset{\Delta}{=} L_{j,\bar{d}}^{(i)} - E\{\delta_j^{(i)}(k) \,|\, \bar{d}\} \; ; \; k=i,3, \; i=1,2, \; j \geq 1$$

From the characteristics of each local LSRAA, (see [4] and section IV of this paper), we obtain:

$$E\{l(\lambda x + \mu y) \,|\, x,y\} > E\{l(\lambda x + \mu y) \,|\, x,y\} - x \geq$$

$$\geq E\{l(\lambda x + \mu y) \,|\, x,y\}[1 - \lambda^{-1}\sup_x \frac{x}{E\{l(x)\}}] =$$

$$= E\{l(\lambda x + \mu y) \,|\, x,y\}[1 - 0.43\lambda^{-1}] \; ; \; \text{for } \lambda > 0 \tag{A.2}$$

From (A.1) and (A.2), we thus obtain:

$$(\lambda_i + \lambda_3) \, L_{1,\bar{d}}^{(i)} > \lambda_i E_{1,\bar{d}}^{(i)} + \lambda_3 E_{1,\bar{d}}^{(i)}(3) \geq$$

$$\geq [\lambda_i + \lambda_3 - 1.29] L_{1,\bar{d}}^{(i)} \; ; \; i=1,2$$

$$(\lambda_i + \lambda_3) \, L_{j,\bar{d}}^{(i)} > \lambda_i E_{j,\bar{d}}^{(i)} + \lambda_3 E_{j,\bar{d}}^{(i)}(3) \geq \tag{A.3}$$

$$\geq [\lambda_i + \lambda_3 - 0.86] L_{j,\bar{d}}^{(i)} \; ; \; i=1,2, \; j \geq 2$$

In addition,

$$AV(\bar{d}) = \simeq \left\{ \lambda_1 \sum_{j=1}^{N_1} E_{j,\bar{d}}^{(1)}(1) + \lambda_2 \sum_{j=1}^{N_2} E_{j,\bar{d}}^{(2)}(2) + \right.$$

$$\left. + \lambda_3 \sum_{j=1}^{N_t} E_{j,\bar{d}}^{(1)}(3) + \lambda_3 \sum_{j=1}^{N_2} E_{j,\bar{d}}^{(2)}(3) \mid \bar{d} \right\} \tag{A.4}$$

From (A.3) and (A.4), we thus conclude, for $\sum_{j=1}^{3} \lambda_j < 1$:

$$\mid AV(\bar{d}) \mid \leq \max(0.43, \lambda_1 + \lambda_3) \, E\{ \sum_{j=1}^{N_1} L_{j,\bar{d}}^{(1)} \mid \bar{d} \} +$$

$$+ \max(0.43, \lambda_2 + \lambda_3) \, E\{ \sum_{j=1}^{N_2} L_{j,\bar{d}}^{(2)} \mid \bar{d} \} <$$

$$< E\{ \sum_{j=1}^{N_1} L_{j,\bar{d}}^{(1)} \mid \bar{d} \} + E\{ \sum_{j=1}^{N_2} L_{j,\bar{d}}^{(2)} \mid \bar{d} \} \tag{A.5}$$

; where,

$$E\{ T_{n+1}^{(s)} - T_n^{(s)} \mid \bar{d}_n = \bar{d} \} = E\{ \sum_{j=1}^{N_1} L_{j,\bar{d}}^{(1)} \mid \bar{d} \} = E\{ \sum_{j=1}^{N_2} L_{j,\bar{d}}^{(2)} \mid \bar{d} \} \tag{A.6}$$

We note, that for $\bar{d} \to \infty$, $\delta_j^{(i)}(k) = \Delta$, $\forall$ i, j, k, and we then obtain directly from (A.1) and (A.4):

$$\lim_{d \to \infty} AV(\bar{d}) =$$

$$= (\lambda_1 + \lambda_3) \left\{ E\{ l(\Delta[\lambda_1 + \frac{\lambda_3}{2}]) \} - \Delta \right.$$

$$+ \left[E\{ l(\Delta[\lambda_1 + \lambda_3]) \} - \Delta \right] \left[\lim_{d \to \infty} E\{ N_1 \mid \bar{d} \} - 1 \right] \right\}$$

$$+ (\lambda_2 + \lambda_3) \left\{ E\{ l(\Delta[\lambda_2 + \frac{\lambda_3}{2}]) \} - \Delta \right.$$

$$+ \left[E\{ l(\Delta[\lambda_2 + \lambda_3]) \} - \Delta \right] \left[\lim_{d \to \infty} E\{ N_2 \mid \bar{d} \} - 1 \right] \right\} \tag{A.7}$$

; where,

$$E\{ l(\Delta[\lambda_1 + \frac{\lambda_3}{2}]) \} + E\{ l(\Delta[\lambda_1 + \lambda_3]) \} \{ \lim_{d \to \infty} E\{ N_1 \mid \bar{d} \} - 1] =$$

$$= E\{l(\Delta[\lambda_2 + \frac{\lambda_3}{2}])\} + E\{l(\Delta[\lambda_2 + \lambda_3])\}[\lim_{d\to\infty} E\{N_2 | \overline{d}\} - 1] \qquad \text{(A.8)}$$

We now state and prove a lemma.

<u>Lemma A</u>

For all given λ_i, i=1,2,3:

$$P\left[\sum_{j=1}^{N_i} L_{j,\overline{d}}^{(i)} \le x | \overline{d}\right] \le P\left[\sum_{j=1}^{N_i} L_{j,\overline{d}}^{(i)} \le x | \overline{d}'\right] ; \text{ for all x, for all } \overline{d} > \overline{d}', \text{ for i=1,2} \qquad \text{(A.9)}$$

Thus,

$$E\left\{\sum_{j=1}^{N_i} L_{j,\overline{d}}^{(i)} | \overline{d}\right\} \ge E\left\{\sum_{j=1}^{N_i} L_{j,\overline{d}}^{(i)} | \overline{d}'\right\} , \text{ for all } \overline{d} > \overline{d}', \text{ for i=1,2} \qquad \text{(A.10)}$$

And,

$$|AV(\overline{d})| < 2\lim_{\overline{d}\to\infty} E\left\{\sum_{j=1}^{N_1} L_{j,\overline{d}}^{(1)} | \overline{d}\right\} \qquad \text{(A.11)}$$

<u>Proof</u>

Let $\overline{d} = \{d^{(j)}\}_{1\le j\le 3}$. Given $d^{(1)}$ and $d^{(3)}$, consider k CRIs generated by the LSRAA in cluster 1. Let l_j denote the length of the j-th such CRI, and let those lengths be temporarily fixed. Let $N(\sum_{i=1}^{k} l_i)$ denote the number of CRIs generated by the LSRAA in cluster 2, within $\sum_{i=1}^{k} l_i$ slots. Then, since $P(l(\lambda d_1) \le x) \ge P(l(\lambda d_2) \le x)$, if $d_2 > d_1$, we easily conclude that $P(N(\sum_{i=1}^{k} l_i) \le x | d^{(2)} = w, d^{(3)}) \ge P(N(\sum_{i=1}^{k} l_i) \le x | d^{(2)} = y, d^{(3)})$, for y < w. Since the latter inequality is true for all k and $\{l_i\}$, and since $P(l_i \le x | \overline{d}) \le P(l_i \le x | \overline{d}')$; for all l_i, x, and $\overline{d} > \overline{d}'$, we conclude (A.9). (A.10) is clearly deduced by (A.9), and (A.11) results from (A.10), in conjunction with (A.5) and (A.6). $\square$

Due to Lemma A, and (A.11) in particular, to prove (12) in Lemma 1, it suffices to show that the limit in (A.11) is bounded, for all $\{\lambda_j\}_{1\le j\le 3}$ such that $\sum_{j=1}^{3} \lambda_j \le 1$. We will perform the latter study later. At this point, we will state and prove a proposition, which is related to part (ii) of Lemma 1.

<u>Proposition A</u>

Given $\{\lambda_j\}_{1\le j\le 3}$, such that $\lambda \overset{\Delta}{=} \sum_{j=1}^{3} \lambda_j < 1$, given some bounded natural number l, such that $\lambda l/\Delta > 1$, let us define,

$$H_2 \overset{\Delta}{=} \{\overline{d} \overset{\Delta}{=} [d^{(j)}]_{1\le j\le 3} : \sum_{j=1}^{3} \lambda_j d^{(j)} \le \lambda l/\Delta\} \qquad \text{(A.12)}$$

$$P_{\overline{d},\lambda,l} \overset{\Delta}{=} \sum_{\overline{e}:\, V(\overline{e}) < V(\overline{d}) - \lambda l\Delta} p_{\overline{d},\overline{e}} \tag{A.13}$$

; where $p_{\overline{d},\overline{e}}$ is as in (5). Consider $F_{\overline{d},V}(z)$ in (6). Then,

$$F_{\overline{d},V}(z) \ge -l\Delta - P_{\overline{d},\lambda,l} \sum_{\overline{e}:\, V(\overline{d}) - V(\overline{e}) > \lambda l\Delta} [V(\overline{d}) - V(\overline{e})] p_{\overline{d},\overline{e}}$$

; for all z in $[0,1)$, and all $\overline{d}$ in $C-H_2$ \tag{A.14}

Proof

From (6), we obtain:

$$F_{\overline{d},V}(z) \ge (1-z)^{-1}\, z^{V(\overline{d})} \left\{ -[1-P_{\overline{d},\lambda,l}]\frac{z^{\lambda l\Delta}-1}{z^\lambda-1} \cdot \frac{z^\lambda-1}{z-1} + \right.$$

$$\left. + \frac{z^{\lambda l\Delta}}{1-z}\left[P_{\overline{d},\lambda,l} - \sum_{\overline{e}:\, V(\overline{e})-V(\overline{d}) < -\lambda l\Delta} z^{V(\overline{e})-V(\overline{d})}\, p_{\overline{d},\overline{e}} \right] \right\} \tag{A.15}$$

Noting that $(1-z)^{-1}(1-z^\lambda) \le 1$, for all $z<1$ and $\lambda<1$, and that $(z-1)^{-1}(z^a-1) \le a$, for all $z<1$ and $a>1$, we obtain from (A.15), the inequality in (A.14). $\square$

Let us now state and prove another lemma.

Lemma B

Given $\{\lambda_j\}_{1\le j\le 3}$, such that $\lambda \overset{\Delta}{=} \sum_{j=1}^{3} \lambda_j < 1$, then, for any finite natural number l, and the resulting then finite subset H_2 as in (A.12), we have:

$$P_{\overline{d},\lambda,l} \sum_{\overline{e}:\, V(\overline{d})-V(\overline{e}) > \lambda l\Delta} [V(\overline{d})-V(\overline{e})]\, p_{\overline{d},\overline{e}} <$$

$$< \lim_{d\to\infty} P(T_{n+1}^{(s)} - T_n^{(s)} > \lambda\frac{\Delta}{\Delta-1} \mid \overline{d}) \sum_{x>l\frac{\Delta}{\Delta-1}} x\, P(T_{n+1}^{(s)} - T_n^{(s)} = x \mid \overline{d})$$

; for all z in $[0,1)$, and all $\overline{d}$ in C \tag{A.16}

Proof

Consider the variables N_i, $i=1,2$. defined in the beginning of this appendix. Then, we obtain:

$$P(V(\overline{d})-V(\overline{e}) > \lambda x\Delta \mid \overline{d}) < P\left[(\lambda_1+\lambda_3)N_1\Delta + (\lambda_2+\lambda_3)N_2\Delta - \lambda[T_{n+1}^{(s)}-T_n^{(s)}] > \lambda x\Delta \mid \overline{d} \right]$$

$$< P\left[\Delta[N_1+N_2] - \Delta x - [T_{n+1}^{(s)}-T_n^{(s)}] > 0 \mid \overline{d} \right] =$$

$$= P\left[T_{n+1}^{(s)} - T_n^{(s)} < \Delta[N_1 + N_2] - \Delta x \mid \bar{d} \right] \qquad (A.17)$$

; where,

$$T_{n+1}^{(s)} - T_n^{(s)} \geq N_1 + N_2 \qquad (A.18)$$

From (A.17) and (A.18) we thus obtain:

$$P_{\bar{d},\lambda,x} \stackrel{\Delta}{=} P(V(\bar{d}) - V(\bar{e}) > \lambda x \Delta \mid \bar{d}) < P(N_1 + N_2 > x \frac{\Delta}{\Delta - 1} \mid \bar{d}) <$$

$$< P(T_{n+1}^{(s)} - T_n^{(s)} > x \frac{\Delta}{\Delta - 1} \mid \bar{d}) \qquad (A.19)$$

Since (A.19) is true for all x, it also implies,

$$\sum_{\bar{e}:\, V(\bar{d}) - V(\bar{e}) > \lambda/\Delta} [V(\bar{d}) - V(\bar{e})]\, p_{\bar{d},\bar{e}} < \sum_{x > l\frac{\Delta}{\Delta - 1}} x\, P\, P(T_{n+1}^{(s)} - T_n^{(s)} = x \mid \bar{d}) \qquad (A.20)$$

Setting $x = l$ in (A.19) and (A.20) and multiplying, gives the first part of (A.16). Now, due to Lemma A and its proof, we also obtain:

$$P(T_{n+1}^{(s)} - T_n^{(s)} \leq x \mid \bar{d}) \leq P(T_{n+1}^{(s)} - T_n^{(s)} \leq x \mid \bar{d}') ;$$

$$\text{for } \bar{d} > \bar{d}'$$

which gives the last part of (A.16). $\square$

We now express a proposition, which ties the results in lemmata A and B with Lemma 1, in the main part of the paper.

Proposition B

(i)   If $\lim_{\bar{d} \to \infty} E\left\{ \sum_{j=1}^{N_1} L_{j,\bar{d}}^{(1)} \mid \bar{d} \right\} < \infty$, for all $\{\lambda_j\}_{1 \leq j \leq 3}$ such that $\sum_{j=1}^{3} \lambda_j < 1$, then (12) in Lemma 1 is satisfied.

(ii)   If there exist finite integer l_o and finite positive constant B', such that ,

$$\lim_{\bar{d} \to \infty} P(T_{n+1}^{(s)} - T_n^{(s)} > l \frac{\Delta}{\Delta - 1} \mid \bar{d}) \sum_{x > l\frac{\Delta}{\Delta - 1}} x\, P\, (T_{n+1}^{(s)} - T_n^{(s)} = x \mid \bar{d}) \leq B', \; \forall l \geq l_o, \; \forall \{\lambda_j\}_{1 \leq j \leq 3} : \sum_{j=1}^{3} \lambda_j < 1 \quad (A.21)$$

then, (13) in Lemma 1 is satisfied with $B = l_o \Delta + B'$ and H_2 as in (A.12), with $l = l_o$. $\square$

The proposition is a direct result of the statements in Lemmata A. and B. Let us now consider the variables $N_i(\{\lambda_j\}, \Delta)$, $i = 1,2$, defined in Section IV.1 of the paper, and the generalized drifts $A^{(i)}V(\bar{d}_\Delta, \{\lambda_j\})$, $i = 1,2$, and $AV(\bar{d}_\Delta, \{\lambda_j\})$, in (14) and (15) respectively. Towards the proof of Theorem 1, in Section IV.1, we then state and prove a lemma.

Lemma C

Let the variables $N_i(\{\lambda_j\}, \Delta)$, $i = 1,2$, be such that:

There exist, finite natural number n_o, ρ: $0<\rho<1$, and finite positive constant c, such that:

$$P(N_i(\{\lambda_j\}, \Delta)=n) < c\rho^n, \ \forall n>n_o, \ \forall \{\lambda_j\}: \sum_{j=1}^{3} \lambda_j <1, \ i=1,2 \qquad (A.22)$$

Then,

(i) If the conditions (17) in Theorem 1 are satisfied for some $\varepsilon>0$, then, there exists a finite subset H_1 of C, and some $\varepsilon'>0$, so that (9) is satisfied. That is, the Markov chain $\{S_n\}_{n\geq0}$ is then ergodic.

(ii) If at least one of the two-generalized drifts $A^{(i)}V(\bar{d}_\Delta, \{\lambda_j\})$, i=1,2, is nonnegative, then there exists a proper subset H of C, so that (11) is then satisfied. That is, the Markov Chain $\{S_n\}_{n\geq0}$ is then nonergodic.

<u>Proof</u>

(i) Let (A.22) be true, and let $\sum_{j=1}^{3} \lambda_j <1$. Define then, $\alpha \stackrel{\Delta}{=} \sup_{\lambda<1} | E_\lambda\{l|\Delta\}-\Delta |$. Given ε in (17), select l_o such that $l_o > n_o$ and $\alpha c \sum_{n\geq l_o+1} \rho^n = \frac{\varepsilon}{2} = (1-\rho)^{-1} \alpha c \rho^{l_o+1}$; that is, select $l_o = \max(n_o+1, [ln\rho]^{-1} ln(\varepsilon[1-\rho][2\alpha c]^{-1} -1))$. Select then, $H_1 \stackrel{\Delta}{=} \{\bar{d} \stackrel{\Delta}{=} [d^{(j)}]_{1\leq j\leq 3} : d^{(1)}\leq l_o, d^{(2)}\leq l_o, d^{(3)}\leq 2l_o\}$. Then, due to (17), and in conjunction with (A.9) in Lemma A we conclude: $AV(\bar{d})<-\frac{\varepsilon}{2}$, for all $\bar{d}$ in $C-H_1$. The proof of part (i) in the Lemma is now complete.

(ii) Let (A.22) be true, and let $\sum_{j=1}^{3} \lambda_j <1$. Let $A^{(i)}V(\bar{d}_\Delta, \{\lambda_j\})\geq0$, for either i=1 or i=2. If then, there existed H_1 subset of C, defined as in the proof of part (i), such that $AV(\bar{d})<-\varepsilon$, for all $C-H_1$ and some $\varepsilon>0$, then due to (A.22), the constant l_o in H_1 could increase (remaining finite), to give $A^{(i)}V(\bar{d}_\Delta, \{\lambda_j\})<0$. Thus, part (ii) of the Lemma is true. $\square$

As it is apparent from the statements of Proposition B and Lemma C, to complete the proofs of Lemma 1 and Theorem 1, in Section IV.1, we need to study the statistical behavior of the variables involved in the description of the time interval $[T_n^{(s)}, T_{n+1}^{(s)}]$, when the lags at $T_n^{(s)}$ are sufficiently long, so that all CRIs in $[T_n^{(s)}, T_{n+1}^{(s)}]$ resolve arrival intervals of length Δ, from all the three subsystems. We perform such studies below.

VI.2. <u>Studies for Sufficiently Long Initial Lags</u>

Our objective here is to study the behavior of the variables $N_i(\{\lambda_j\}, \Delta)$, i=1,2, $A^{(i)}V(\bar{d}_\Delta, \{\lambda_j\})$, i=1,2, and $AV(\bar{d}_\Delta, \{\lambda_j\})$, defined in Section IV.1. For tractability, we introduce simpler notation. Give $\{\lambda_j\}_{1\leq j\leq 3}$, we first define:

$$x_i \stackrel{\Delta}{=} (\lambda_i+\lambda_3)\Delta , \ i=1,2$$

$$y \underset{=}{\Delta} (\lambda_3 \Delta) / 2 \qquad\qquad (A.23)$$

$$\mu_i \underset{=}{\Delta} \lambda_i + \lambda_3 \; , \; i=1,2$$

For the rest of this subsection, we assume that every CRI, within the interval $[T_n^{(s)}, T_{n+1}^{(s)}]$, resolves arrivals in a window of length Δ, from all the relative subsystems; that is, a CRI generated by the algorithm in cluster i, resolves a Δ-length window from subsystem i, and a Δ-length window from subsystem 3. We then define:

i=1,2, ; $P_i(e,m)$:

The probability that $T_n^{(s)}+e$ is a collision resolution point for the LSRAA in cluster i, and that it is the m-th such point. We note that m=1 refers to the end of the first CRI, which starts at $T_n^{(s)}$.

i=1,2 ; $P^{(i)}(e,m)$:

The probability that $T_{n+1}^{(s)} - T_n^{(s)} = e$, and there are m CRIs for the algorithm in cluster i, in $[T_n^{(s)}, T_{n+1}^{(s)}]$.

i=1,2 ; $P^{(i)}(m)$:

The probability that there are m CRIs for the algorithm in cluster i, in $[T_n^{(s)}, T_{n+1}^{(s)}]$.

l_x:

The length of a CRI, when it resolves an interval of length d containing arrivals from an intensity λ Poisson process, where λ and d are such that, $\lambda d = x$.

$0 \le n \le k$; $l_{n,k-n}$:

The number of slots needed by either one of the two LSRAAs in the system to transmit k packets, when n of them have counter values equal to one, and the remaining k-n packets have counter values equal to two.

$\left. \begin{array}{c} 0 \le l \le e-1 \\ i=1,2 \end{array} \right\}$; $P_i(e,m;l,n)$:

The probability that $T_n^{(s)}+e$ is a collision resolution point for the LSRAA in cluster i, that it is the m-th such point, that $T_{n+1}^{(s)} - T_n^{(s)} > e$, that the last before $T_n^{(s)}+e$ collision resolution point for the LSRAA in the other cluster occurs at $T_n^{(s)}+l$, and that it is the n-th such point.

i, = 1,2 ; $P_{i,i_c}(e,m;e,n)$:

$i_c = \begin{cases} 1, & \text{if } i=2 \\ 2, & \text{if } i=1 \end{cases}$

The probability that $T_{n+1}^{(s)} = T_n^{(s)}+e$, and that this point is the m-th collision resolution point for the algorithm in cluster i, and it is the n-th such point for the algorithm in cluster i_c.

To start with, the deployed LSRAAs induce the following recursions, regarding the lengths $l_{n,k-n}$ (see [4]):

$$P(l_{1,k-1} = m) = P(l_{k-1,0} = m-1)$$

$$P(l_{0,k}=m) = P(l_{k,0} = m-1)$$

$$P(l_{1,0} = 1) = P(l_{0,0} = 1) = P(l_{0,1} = 2) = P(l_{1,1} = 2) = 1$$

$$\left.\begin{array}{l} k\geq 2 \\ m\geq k+1 \end{array}\right\} \; ; P(l_{2,k-2}=m) = 2^{-2}\left\{ P(l_{k,0}=m-2)+P(l_{2,k-2}=m-1)+2P(l_{k-1,0})=m-2)\right\}$$

$$\left.\begin{array}{l} k\geq n \\ n\geq 3 \\ m\geq 2k-1 \end{array}\right\} \; ; P(l_{n,k-n}=m) = 2^{-n}\left\{ P(l_{k,0}=m-2)+P(l_{n,k-n}=m-1)+nP(l_{k-1,0}=m-2)+ \right.$$

$$\left. + \sum_{i=2}^{n-1} \binom{n}{i} P(l_{i,k-i}=m-1)\right\} \tag{A.24}$$

In addition, for $\lfloor\;\rfloor$ denoting integer part, we have:

$$P(l_x=m)= \sum_{0\leq k\leq \lfloor \frac{m+1}{2}\rfloor} e^{-x}\frac{x^k}{k!} P(l_{k,0}=m) \tag{A.25}$$

We also have the following recursions and expressions:

$$\left.\begin{array}{l} i=1,2 \\ e\geq m \end{array}\right\} \; ; P_i(e,m) = \left\{\begin{array}{l} P(l_{x_i-y}=e) \; ; \; \text{if } m=1 \\ \sum_{k=m-1}^{e-1} P_i(k,m-1)P(l_{x_i}=e-k) \; ; \; \text{if } m\geq 2 \end{array}\right. \tag{A.26}$$

For,

$$i_c \stackrel{\Delta}{=} \left\{\begin{array}{l} 1 \text{ , if } i=2 \\ 2 \text{ , if } i=1 \end{array}\right. \quad U(x) \stackrel{\Delta}{=} \left\{\begin{array}{l} 1, \text{ if } x\geq 0 \\ 0, \text{ if } x<0 \end{array}\right. \tag{A.27}$$

we also have:

$$i=1,2, \; m\geq 1, \; e\geq m; \; P_i(e,m;0,0) = P_i(e,m)P(l_{x_{i_c}-y}>e) \tag{A.28}$$

$$i=1,2, m\geq 1, k\geq 1, e\geq m; P_i(e+k,1;e,m)=P_{i_c}(e,m;0,0)P(l_{x_{i_c}}>k)P(l_{x_i-y}=e+k)P^{-1}(l_{x_{i_c}-y}>e) \tag{A.29}$$

$$\left.\begin{array}{l} m\geq 2, \; e\geq m \\ i=1,2 \end{array}\right\} \; ; P_{i,i_c}(e,m;e,1)=P_i(e,m)P(l_{x_{i_c}-y}=e) \tag{A.30}$$

$$\left.\begin{array}{l} m\geq 2 \\ n\geq 1 \\ e\geq m \\ l\geq n \\ i=1,2 \end{array}\right\} \; ; P_i(e,m;l,n) = \sum_{k=1}^{min(e-l-1,e-m+1)} P_i(e-k,m-1;l,n)P(l_{x_i}=k)P(l_{x_{i_c}}>e-l)P^{-1}(l_{x_{i_c}}>e-k-l) +$$

$$+U(l-m)\sum_{k=e-l+1}^{e+1-m} P_{i_c}(l,n;e-k,m-1)P(l_{x_i}=k)P(l_{x_{i_c}}>e-l)P^{-1}(l_{x_i}>l+k-e) \tag{A.31}$$

$$\left.\begin{array}{l} m\geq 2 \\ n\geq 2 \end{array}\right\} ; P_{i,i_c}(e,m;e,n) = \sum_{l=n+1}^{e-1} \sum_{k=1}^{min(e-l-1,e+1-m)} P_i(e-k,m-1;l,n-1)P(l_{x_i}=k).$$

$$.P(l_{x_{i_c}}=e-l)P^{-1}(l_{x_{i_c}}>e-k-l)$$

$$+ \sum_{l=m-1}^{e-1} \sum_{k=1}^{min(e-l-1,e+1-n)} P_{i_c}(e-k,n-1;l,m-1)P(l_{x_{i_c}}=k)P(l_{x_i}=e-l)P^{-1}(l_{x_i}>e-k-l) \quad (A.32)$$

$$P^{(i)}(e,m) = \sum_n P_{i,i_c}(e,m;e,n) \; ; \; i=1,2 \quad (A.33)$$

$$P^{(i)}(m) = \sum_{e\geq m} P^{(i)}(e,m) \; ; \; i=1,2 \quad (A.34)$$

Referring to the expressions in (14) in Section IV.1 of the paper, and from (A.33), we also obtain:

$$i=1,2; \; A^{(i)}V(\overline{d}_\Delta,\{\lambda_j\}) = \sum_{m\geq 1}\sum_{e\geq m} P^{(i)}(e,m)[e-m\Delta]$$

$$= \sum_{m\geq 1}\sum_{e\geq m} P^{(i)}(e,m)[e-m\frac{x_i}{\mu_i}] \quad (A.35)$$

Subject to the satisfaction of Lemma 1 and Theorem 1, in Section IV.1, the $\{\lambda_j\}_{1\leq j\leq 3}$ values for which the Markov Chain $\{S_n\}_{n\geq 0}$ is ergodic, are found as those that make the expressions in (A.35) negative.

Via some tedious manipulation of the expressions found in this section, which we do not include here, we found the result stated in the proposition below.

Proposition C

Given $\epsilon>0$, there exist positive integer m_0 and $0<\mu<1$, such that:

$$i=1,2 \; ; \; \sum_{l\geq m+1} P^{(i)}(l) < \epsilon + \mu \sum_{l\geq m} P^{(i)}(l)$$

$$\forall m>m_0 \; , \; \forall \{\lambda_j\}_{1\leq j\leq 3} \; : \; \sum_{j=1}^{3} \lambda_j < 1 \quad (A.36) \quad \Box$$

We are now ready to complete the proofs of Lemma 1 and Theorem 1, in Section IV.1. We do this, in Section VI.3 below.

VI.3 The Proofs of Lemma 1 and Theorem 1

From Proposition C and expression (A.36), we conclude that there exist constants n_0, ρ, and c, such that (A.22) in Lemma C is satisfied. Thus, the statements in the latter Lemma hold. In addition, due to (A.22), (A.21) in Proposition B clearly holds as well then, and also $\lim_{d\to\infty} E\{\sum_{j=1}^{N_t} L_{j,d}^{(1)} | \overline{d}\}<\infty$, for all $\{\lambda_j\}_{1\leq j\leq 3}$ such that $\sum_{j=1}^{3} \lambda_j < 1$. Thus, the statements in Proposition B and in Lemma C all hold, and so do Lemma 1 and Theorem 1.

VI.4 Recursions and Systems for Delay Analysis

Here, we present the quantities needed for the computation of the expected per packet delays, and their recursions. Let us define, for i_c as in (A.27):

$(d_i, \delta_3; \{l, d_{i_c}, d_3\})$:
  $; i=1,2$

During the process of the algorithmic system, the event of being at some time instant t when the first slot of a CRI for the LSRAA in cluster i begins, that at t the lag for subsystem i is d_i and the lag for subsystem 3 is δ_3, that the last before t CRI for subsystem i_c starts at t-l, and that at t-l the lag for subsystem i_c is d_{i_c} and the lag for subsystem 3 is d_3.

$h_i(d_i, \delta_3; \{l, d_{i_c}, d_3\})$:
  $; i=1,2$

Given the event $(d_i, \delta_3; \{l, d_{i_c}, d_3\})$, the length of the time interval until the first after that occurence of some $T_n^{(s)}$ with S_n equal to the unity vector.

$\psi_i^{(j)}(d_i, \delta_3; \{l, d_{i_c}, d_3\})$:
  $i=1,2, j=1,2,3$

Given the event $(d_i, \delta_3; \{l, d_{i_c}, d_3\})$, the expected cumulative delay of all the packets from subsystem j that are transmitted from the instant when the above event occurs, to the instant when the first after that $T_n^{(s)}$ with S_n equal to the unity vector occurs.

$A_i^{(j)}(d_i, \delta_3; \{l, d_{i_c}, d_3\})$:
  $; i=1,2, j=1,2,3$

Given the event $(d_i, \delta_3; \{l, d_{i_c}, d_3\})$, the expected cumulative delay that those packets from subsystem j, which are waiting at the point of the event, have already experienced.

$\theta_i^{(j)}(d_i, \delta_3; \{l, d_{i_c}, d_3\})$:
  $; i=1,2, j=i,3$

Given the event $(d_i, \delta_3; \{l, d_{i_c}, d_3\})$, the expected cumulative waiting time of those packets from subsystem j which are transmitted during the CRI of the LSRAA in cluster i that starts with the event.

$w_i^{(j)}(d_i, \delta_3; \{l, d_{i_c}, d_3\})$:
  $i=1,2, j=1,3$

Given the event $(d_i, \delta_3; \{l, d_{i_c}, d_3\})$, the expected cumulative transmission time of these packets from subsystem j which are transmitted during the CRI of the LSRAA in cluster i that starts with the event.

$Z_{n,k-n}$:
  $; n\geq 0, k\geq n$

Given that during some CRI from any of the two LSRAAs, n packets have counter values equal to 1 and k-n packets hae counter values equal to 2, the expected cumulative delay of all the packets transmitted from the point of the (n, k-n) occurence to the end of the CRI.

To start with, the operations of each of the two LSRAAs give rise to the following recursions:

$$Z_{0,0} = 0, Z_{1,0} = 1, Z_{0,k} = k + Z_{k,0}; k\geq 1$$

$$Z_{1,k-1} = k + Z_{0,k-1} = 2k-1 + Z_{k-1,0} \; ; \; k \geq 2 \qquad (A.37)$$

$$Z_{n,k-n} = k + 2^{-n} \sum_{i=0}^{n} \binom{n}{i} Z_{i,k-i} \; ; \; n \geq 2$$

Then, defining

$$v_3 = \begin{cases} \lambda_3 \; ; \; \text{if } l \geq 1 \\[2mm] \dfrac{\lambda_3}{2} \; ; \; \text{if } l = 0 \end{cases} \qquad (A.38)$$

$$\zeta_j = \begin{cases} \lambda_i \min(d_i, \Delta) \; ; \; \text{if } j{=}i \\[2mm] v_3 \min(\delta_3, \Delta) \; ; \; j{=}3 \end{cases} \qquad A.39$$

we have:

$$i = 1,2 \; ; \; w_i^{(j)}(d_i, \delta_3 \, ; \{l, d_{i_e}, d_3\}) = w_i^{(j)}(d_i, \delta_3) =$$

$$\zeta_j [\lambda_i \min(d_i, \Delta) + v_3 \min(\delta_3, \Delta)]^{-1} \cdot \exp\{-\lambda_i \min(d_i, \Delta) - v_3 \min(\delta_3, \Delta)\}$$

$$\cdot \sum_{k \geq 0} \frac{[\lambda_i \min(d_i, \Delta) + v_3 \min(\delta_3, \Delta)]^k}{k!} Z_{k,0} \qquad (A.40)$$

In addition,

$$i{=}1,2 \; ; \; \theta_i^{(i)}(1, \delta_3; \{l, d_{i_e}, d_3\}) = A_i^{(i)}(1, \delta_3 \, ; \{l, d_{i_e}, d_3\}) = 1.5\lambda_i \; ; \; \forall \, \delta_3, \{l, d_{i_e}, d_3\}$$

$$i = 1,2 \; ; \; \theta_i^{(3)}(d_i, 1; \{l, d_{i_e}, d_3\}) = A_i^{(3)}(d_i, 1; \{l, d_{i_e}, d_3\}) = 1.5\lambda_3 \; ; \; \forall \, l \geq 1 \qquad (A.41)$$

$$\theta_i^{(j)}(d_i, \delta_3; \{0, d_{i_e}, d_3\}) = A_i^{(j)}(d_i, \delta_3; \{0, d_{i_e}, d_3\}) = 0, \text{ for } \delta_3 \neq d_3$$

$$A_1^{(j)}(d_i, d_3; \{0, d_2, d_3\}) = A_2^{(j)}(d_2, d_3; \{0, d_1, d_3\}); \; j{=}1,2,3$$

$$i = 1,2 \; ; \; A_i^{(3)}(d_i, 1; \{0, d_{i_e}, 1\}) = 2\,\theta_i^{(3)}(d_i, 1; \{0, d_{i_e}, 1\}) = 1.5\lambda_3 \; ; \; \forall \, d_i, d_{i_e}$$

$$i{=}1,2 \; ; \; A_i^{(i)}(d_i, \delta_3; \{l, d_{i_e}, d_3\}) = \theta_i^{(i)}(d_i, \delta_3; \{l, d_{i_e}, d_3\}) \; ; \; \forall \, d_i \leq \Delta, \; \forall \, \delta_3, \{l, d_{i_e}, d_3\}$$

$$i{=}1,2 \; ; \; \theta_i^{(3)}(d_i, \delta_3; \{l, d_{i_e}, d_3\}) = \begin{cases} A_i^{(3)}(d_i, \delta_3; \{l, d_{i_e}, d_3\}) \; ; \; \forall \, \delta_3 \leq \Delta, \text{ if } l \geq 1 \\[2mm] 2^{-1} A_i^{(3)}(d_i, \delta_3; \{l, d_{i_e}, d_3\}) \; ; \; \forall \, \delta_3 \leq \Delta, \text{ if } l = 0 \end{cases}$$

$$i{=}1,2 \; ; \; A_i^{(i_e)}(d_i, \delta_3; \{l, 1, d_3\}) = \begin{cases} 1.5\lambda_{i_e} \; ; \; \text{if } l = 0 \\[2mm] l(\dfrac{l}{2}+1)\lambda_{i_e} \; ; \; \text{if } l \geq 1, \; \forall \, d_i, \delta_3, d_3 \end{cases}$$

For v_3 as in (A.39) and ζ_j as in (A.40):

$$\left.\begin{array}{l} i=1,2 \\ j=1,2,3 \end{array}\right\} ; A_i^{(j)}(d_i, \delta_3 ; \{l, d_{i_e}, d_3\}) - \theta_i^{(j)}(d_i, \delta_3 ; \{l, d_{i_e}, d_3\}) =$$

$$A_i^{(j)}(d_i+m-\lambda_i^{-1}\zeta_i, \delta_3+m-\min(\delta_3,\Delta); \{l+m, d_{i_e}, d_3\}) - m[\lambda_j d_j - \zeta_j]$$

$$-\lambda_j m(\frac{m}{2}+1); \text{w.p. } P(l_{\zeta_i+v_3}=m)P(l_{\zeta_{i_e}+v_3}>m+l) \qquad (A.42)$$

$$A_i^{(j)}(d_i+m-\lambda_i^{-1}\zeta_i, \delta_3+m-\min(\delta_3,\Delta); \{0, d_{i_e}+m+l-\lambda_{i_e}^{-1}\zeta_{i_e}, \delta_3+m-\min(\delta_3,\Delta)\})$$

$$-m[\lambda_j d_j - \zeta_i] - \lambda_j m(\frac{m}{2}+1); \text{w.p. } P(l_{\zeta_i+v_3}=m)P(l_{\zeta_{i_e}+v_3}=m+l)$$

$$A_{i_e}^{(j)}(d_{i_e}+m-\lambda_{i_e}^{-1}\zeta_{i_e}, d_3+m-\min(d_3,\Delta); \{m-l, d_i, \delta_3\}) - (m-l)[\lambda_j \zeta_j - d_j]$$

$$-\lambda_j(m-l)[\frac{m-l}{2}+1], \text{ for } m>l; \text{w.p. } P(l_{\zeta_{i_e}+v_3}=m)P(l_{\zeta_i+v_3}>m-l)$$

$$h_1(d_1, d_3 ; \{0, d_2, d_3\}) = h_2(d_2, d_3; \{0, d_1, d_3\})$$

For $d_i \leq \Delta, d_{i_e} \leq \Delta, d_3 \leq \Delta$; $h_i(d_i, d_3 ; \{0, d_{i_e}, d_3\}) = 1$; w.p. $P(l_{\zeta_i+\zeta_3}=1)P(l_{\zeta_{i_e}+\zeta_3}=1)$

$i=1,2$; $h_i(d_i, \delta_3 ; \{l, d_{i_e}, d_3\}) =$

$$m + h_i(d_i+m-\lambda_i^{-1}\zeta_i, \delta_3+m-\min(\delta_3,\Delta); \{l+m, d_{i_e}, d_3\})$$

$$; \text{w.p. } P(l_{\zeta_i+v_3}=m) P(l_{\zeta_{i_e}+v_3} > m+l)$$

$$m + h_i(d_i+m-\lambda_i^{-1}\zeta_i, \delta_3+m-\min(\delta_3,\Delta); \{0, d_{i_e}+m+l-\lambda_{i_e}^{-1}\zeta_{i_e}, \delta_3+m-\min(\delta_3,\Delta)\})$$

$$; \text{w.p. } P(l_{\zeta_i+v_3}=m)P(l_{\zeta_{i_e}+v_3}=m+l) \qquad (A.43)$$

$$m + h_{i_e}(d_{i_e}+m-\lambda_{i_e}^{-1}\zeta_{i_e}, d_3+m-\min(d_3,\Delta); \{m-l, d_i, \delta_3\}), \text{ for } m > l,$$

$$; \text{w.p. } P(l_{\zeta_{i_e}+v_3}=m)P(l_{\zeta_i+v_3} > m-l)$$

We note that the expected value H in (18) is such that, $H = h_1(1, 1; \{0,1,1\}) = h_2(1, 1; \{0, 1, 1\})$.

Defining,

$$\theta_i^{(j)}(d_i, \delta_3) = \sum_{\{l, d_{i_e}, d_3\}} \theta_i^{(j)}(d_i, \delta_3; \{l, d_{i_e}, d_3\})P(d_i, \delta_3; \{l, d_{i_e}, d_3\} \mid d_i, \delta_3) \qquad (A.44)$$

we also obtain:

$$i = 1,2 \; \psi_i^{(j)}(1,1;\{0,1,1\}) = \begin{cases} w_i^{(j)}(1,1) + \theta_i^{(j)}(1,1;\{0,1,1\}), \text{ for } j=1,2 \\ \qquad ; \text{w.p. } P(l_{\lambda_1+\lambda_3}|2=1)P(l_{\lambda_2+\lambda_3}|2=1) \\ w_1^{(3)}(1,1) + w_2^{(3)}(1,1) + \\ + \theta_1^{(3)}(1,1;\{0,1,1\}) + \theta_2^{(3)}(1,1;\{0,1,1\}), \text{ for } j=3 \\ ; \text{w.p. } P(l_{\lambda_1+\lambda_3}|2=1)P(l_{\lambda_2+\lambda_3}|2=1) \end{cases} \quad (A.45)$$

$$\psi_i^{(j)}(d_i, \delta_3; \{0, d_{i_e}, d_3\}) = 0, \text{ if } \delta_3 \neq d_3, \; i=1,2, \; j=1,2,3$$

$$\psi_i^{(j)}(d_i, d_3; \{0, d_{i_e}, d_3\}) = \begin{cases} w_i^{(j)}(d_i,d_3) + \theta_i^{(j)}(d_i, d_3; \{0, d_{i_e}, d_3\}) \\ , \text{ for } j=1,2; \text{ w.p. } P(l_{\lambda_i d_i+d_3\lambda_3}|2=1)P(l_{\lambda_{i_e} d_{i_e}+d_3\lambda_3}|2=1) \\ w_i^{(3)}(d_i, d_3) + w_{i_e}^{(3)}(d_{i_e}, d_3) \\ + \theta_i^{(3)}(d_i, d_3, \{0, d_{i_e}, d_3\}) + \theta_{i_e}^{(3)}(d_{i_e}, d_3; \{0,d_i, d_3\}), \\ , \text{ for } j=3; \text{ w.p. } P(l_{\lambda_i d_i+d_3\lambda_3}|2=1)P(l_{\lambda_{i_e} d_{i_e}+d_3\lambda_3}|2=1) \end{cases}$$
$$; \text{ for } d_i \leq \Delta, \, d_{i_e} \leq \Delta, \, d_3 \leq \Delta$$

$$\psi_1^{(j)}(d_1, d_3; \{0, d_2, d_3\}) = \psi_2^{(j)}(d_2, d_3; \{0, d_1, d_3\}); \; j=1,2,3 \quad (A.46)$$

$$\psi_i^{(j)}(d_i,\delta_3;\{l,d_{i_e},d_3\}) = \begin{cases} w_i^{(j)}(d_i,\delta_3) + \theta_i^{(j)}(d_i, \delta_3; \{l, d_{i_e}, d_3\}) \\ + \psi_i^{(j)}(d_i+m-\lambda_i^{-1}\zeta_i, \delta_3+m-\min(\delta_3,\Delta) \\ \qquad ;\{l+m,d_{i_e},d_3\}) \\ ; \text{ w.p. } P(l_{\zeta_i+v_3}=m) \, P(l_{\zeta_{i_e}+v_3} > m+l) \\ w_i^{(j)}(d_i,\delta_3) + \theta_i^{(j)}(d_i, \delta_3;\{l,d_{i_e},d_3\}) \\ + \psi_i^{(j)}(d_i+m-\lambda_i^{-1}\zeta_i, \delta_3+m-\min(\delta_3,\Delta); \\ \{0,d_{i_e}+m+l-\lambda_{i_e}^{-1}\zeta_{i_e}, \delta_3+m-\min(\delta_3,\Delta)\}) \\ ; \text{ w.p. } P(l_{\zeta_i+v_3}=m) \, P(l_{\zeta_{i_e}+v_3}=m+l) \end{cases}$$
$$; i=1,2 \quad j=1,2 \qquad\qquad (A.47a)$$

$$\psi_i^{(j)}(d_i,\delta_3;\{l,d_{i_e},d_3\}) = \begin{cases} \psi_{i_e}^{(j)}(d_{i_e}+m-\lambda_{i_e}^{-1}\zeta_{i_e}, d_3+m-\min(d_3,\Delta); \\ \{m-l,d_i,\delta_3\}) \\ , \text{for } m>l; \text{ w.p. } P(l_{\zeta_{i_e}+v_3}=m)P(l_{\zeta_i+v_3}>m-l) \end{cases}$$
$$; i=1,2 \quad j=1,2 \qquad\qquad (A.47b)$$

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$$\psi_i^{(3)}(d_i,\delta_3;\{l,d_{i_e},d_3\}) = \begin{cases} w_i^{(3)}(d_i,\delta_3) + \theta_i^{(3)}(d_i,\delta_3;\{l,d_{i_e},d_3\}) \\ + \psi_i^{(3)}(d_i+m-\lambda_i^{-1}\zeta_i,\delta_3+m-\min(\delta_3,\Delta);\{l+m,d_{i_e},d_3\}) \\ \quad ; \text{w.p. } P(l_{\zeta_i+v_3}=m) \; P(l_{\zeta_{i_e}+v_3} > m+l) \\ w_i^{(3)}(d_i,\delta_3) + \theta_i^{(3)}(d_i,\delta_3;\{l,d_{i_e},d_3\}) \\ + w_{i_e}^{(3)}(d_{i_e},d_3) + \theta_{i_e}^{(3)}(d_{i_e},d_3) + \\ \psi_i^{(j)}(d_i+m-\lambda_i^{-1}\zeta_i,\delta_3+m-\min(\delta_3,\Delta; \\ \{0,d_{i_e}+m+l-\lambda_{i_e}^{-1}\zeta_{i_e},\delta_3+m-\min(\delta_3,\Delta)\}) \\ \quad ; \text{w.p. } P(l_{\zeta_i+v_3}=m) \; P(l_{\zeta_{i_e}+v_3}=m+l) \end{cases} \qquad \text{(A.48a)}$$

$$\psi_i^{(3)}(d_i,\delta_3;\{l,d_{i_e},d_3\}) = \begin{cases} w_{i_e}^{(3)}(d_{i_e},d_3) + \theta_{i_e}^{(3)}(d_{i_e},d_3) \\ + \psi_{i_e}^{(j)}(d_{i_e}+m-\lambda_{i_e}^{-1}\zeta_{i_e},d_3+m-\min(d_3,\Delta);\{m-l,d_i,\delta_3\}), \\ \quad \text{for } m>l; \\ ;\text{w.p. } P(l_{\zeta_{i_e}+v_3}=m)P(l_{\zeta_i+v_3} > m-l) \end{cases} \qquad \text{(A.48b)}$$

We note that the expected value $W^{(j)}$ in (18) is such that $W^{(j)} = \psi_1^{(j)}(1,1;\{0,1,1\}) = \psi_2^{(j)}(1,1;\{0,1,1\})$, j=1,2,3.

The relationships included in this section induce infinite dimensionality linear systems. We used the methodology in [6] to derive upper and lower bounds on those systems, and subsequently upper and lower bounds on the delays $D^{(j)}$, j=1,2,3.

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